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Assessment of Interaction in Process Control Systems

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Abstract

Interaction between control loops has long been recognised as an area for concern within the field of process control. The first formalised method for assessing the level of interaction present in a system of loops was Bristol's relative gain array. Being based solely on steady state data, construction of the RGA is pleasingly simple and application of the technique has attracted significant researchers. Where strong interaction effects arise as a problem in practice, their elimination can be approached at a progression of levels each carrying a progressively increased cost implication. The decision as to which of these levels of approach is warranted in overcoming any specific case of interaction involves gaining insight into the mechanics of the system itself. In the future, development in the use of detailed dynamic modelling and simulation may lead to the possibilities of evaluating the degree and form of interaction present and of developing compensating strategies.

In this paper we aim at drawing attention to the fact that in considering interaction between feedback control loops an important distinction should be drawn between cases of true (two-way) interaction and cases of what can be termed as "one-way interaction" or "disturbance propagation" from one loop to another.

Key words: Assessment of interaction, Degree of interaction, Multivariable systems, One-way or Two-way decouplers, Disturbance propagation, Relative gain array.

1 Introduction

The first formalised method for assessing the level of interaction present in a system of loops was Bristol's relative gain array (RGA, Bristol [2]). Being based solely on steady state data, the construction of the RGA is pleasingly simple and application of the technique has attracted significant followers (Shinskey [12], McAvoy [11]), Bequette *et al.* [1] and Gagnepain *et al.* [6].

Interaction between control loops has long been recognised as an area for concern within the field of process control. McAvoy [11] as an enthusiastic proponent of the RGA, highlights the fact that its use in interaction analysis is inherently bound to the concept of the "perfect" controller, a controller whose action prevents the appearance of any deviation from set point in its controlled condition following a disturbance.

Poorer results, however, might be expected to arise when real controllers are used whose action is clearly less than "perfect" in allowing at least some deviation from set point to occur. Thus whilst the attractively simple RGA approach gives insight into idealised control system behaviour, it is difficult to envisage a similar degree of insight becoming possible in the case of "real" control without taking into account dynamic aspects. Accordingly, Witcher and McAvoy [14], Bristol [3] and later Tung and Edgar [13] have examined extension of the RGA to take account of dynamic features in the dynamic relative gain array (DRGA).

In addition, a number of other workers have cited shortcomings in specific applications of the RGA and a variety of developments and related approaches have been proposed in order to overcome these supposed shortcomings. A number of these alternatives have been reviewed and evaluated by Jensen *et al.* [7].

The two prime areas of application for interaction analysis are during system design, where controlled and manipulated variables must be paired favourably, and during plant operation, Khelassi *et al.* [9, 10].

Where strong interaction effects arise as a problem in practice, their elimination can be approached at a progression of levels each carrying a progressively increased cost implication.

The decision as to which of these levels of approach is warranted in overcoming any specific case of interaction involves gaining insight into the mechanics of the system itself. In the future, development in the use of detailed dynamic modelling and simulation may lead to the possibilities of evaluating the degree and form of interaction present and of developing compensating strategies. The purpose of this paper is to put forward an analysis technique that compromises in accommodating even simple dynamic models whilst focusing attention more tightly on the precise form of any interaction present. The insight thus gained forms a basis for defining a well tailored compensating strategy, including the choice between one-way or two-way decouplers.

2 Interaction and disturbance propagation

The fundamental structures underlying interactive behaviour between two feedback control loops are shown in Figure 1. In Figure 1a the loops are independent and no changes in either loop can affect the other. In Figure 1b a "one-way" path exists linking one loop to the other but not vice versa. This structure allows propagation of disturbances from the first loop into the second and will therefore be referred to here as a "disturbance propagation" rather than a "one-way interactive" structure. In Figure 1c the possibility exists for interaction in the true sense of the word since the action of either controller is transmitted to the opposite loop through the two interactor transfer functions $G_{12}(S)$ and $G_{21}(S)$. The latter case also embodies the minimal triple loop structure necessary for true interaction (*i.e.*, loop transmittances $-G_{c1}(S)G_{11}(S)$, $-G_{c2}(S)G_{22}(S)$ and $G_{c1}(S)G_{21}(S)G_{c2}(S)G_{12}(S)$).

In principle the introduction of a suitable decoupler will allow reduction of the structures in Figures 1b and 1c to the independent form of Figure 1a. Note, however, that only a "one-way" decoupler is called for in the case of the disturbance propagation structure.

The structures in Figure 1 are of course idealised and real systems will seldom conform rigidly to them in the sense that the interlinking functions whilst present may act only at a very weak level. Most significantly, an interactive system with one strong and one relatively weak interactor, although strictly having the structure in Figure 1c, may reduce effectively to the disturbance propagation form in Figure 1b and hence may call for compensation by means of only a "one-way decoupler".

2.1 The relative gain array (RGA)

Considerable insight into the behaviour of interacting systems under perfect control is afforded by the RGA. For a system of interacting loops the general element of the RGA can be written as

$$\Lambda = (\lambda_{ij}), \tag{1}$$

where

$$(\lambda_{ij}) = \frac{\left(\dfrac{\partial \theta_i}{\partial u_j}\right)_{\text{all loops open}}}{\left(\dfrac{\partial \theta_i}{\partial u_j}\right)_{\text{only } \theta_i - u_j \text{ loop open}}} \tag{2}$$

and the partial derivatives represent steady state gain factors evaluated under the conditions noted in parenthesis (the array therefore relates to the use of perfect controllers only). For a system of two loops application of the property that elements in any row or column in Λ sum to unity allows construction of the full array on the basis of (λ_{11}) only. Having constructed Λ, an extensive body of experience in its interpretation can be drawn upon (see for example McAvoy [11]). In essence, diagonal dominance in Λ with diagonal elements close to unity indicates both correct pairing of measured and manipulated variables and the absence of potential for interaction.

2.2 The dynamic relative magnitude array (DRMA)

The RGA has been extended to take account of system dynamics in the DRGA but this again relates to use of perfect controllers. The effects of using real controllers in a system can be investigated instead on the basis of a similar dimensionless dynamic array criterion which in the Laplace domain we define as follows for the case of a two-loop system:

$$\Delta = (\delta_{ij}), \tag{3}$$

where

$$\delta_{11} = \frac{\left(\dfrac{\theta_1(s)}{\theta_{1sp}(s)}\right)_{oo}}{\left(\dfrac{\theta_1(s)}{\theta_{1sp}(s)}\right)_{oc}}, \qquad \delta_{12} = \frac{\left(\dfrac{\theta_1(s)}{\theta_{2sp}(s)}\right)_{cc}}{\left(\dfrac{\theta_2(s)}{\theta_{2sp}(s)}\right)_{cc}},$$
$$\delta_{21} = \frac{\left(\dfrac{\theta_2(s)}{\theta_{1sp}(s)}\right)_{cc}}{\left(\dfrac{\theta_2(s)}{\theta_{2sp}(s)}\right)_{cc}}, \qquad \delta_{22} = \frac{\left(\dfrac{\theta_2(s)}{\theta_{2sp}(s)}\right)_{oo}}{\left(\dfrac{\theta_2(s)}{\theta_{2sp}(s)}\right)_{oc}}. \tag{4}$$

Subscripts here refer to the status of the two control loops shown in Figure 1c, where loops are broken by disconnecting the measurement signal (*i.e.*, OC indicates loop 1 open and loop 2 closed, whilst CO indicates loop 1 closed and loop 2 open).

Since we are primarily interested in the relative size of movements in the variables θ_1 and θ_2, an examination of the magnitudes of the elements of Δ in the frequency plane is especially revealing. Calculation of these magnitudes on the basis of the underlying system transfer functions is straightforward and plots in the magnitude versus log frequency format are most appropriate. In this form we refer here to Δ as the Dynamic Relative Magnitude Array (DRMA), Khelassi, [8].

Inspection of the diagonal elements Δ shows that the forward path term in both numerator and denominator involves the real controller transfer function, which as a result cancels out. If, in addition, integral action is present in each controller (thus enforcing the equivalent of perfect control at the steady state), then the low frequency asymptotes of magnitude (*i.e.*, low frequency values of the DRMA diagonal elements) are identical to the corresponding diagonal entries in the RGA. The DRMA diagonal can be viewed as an extension of the corresponding RGA elements across the frequency spectrum with similar interpretations being placed on values.

The off-diagonal elements in Δ have been chosen so as to highlight the propensity of the system for disturbance propagation. In particular, use of the same denominator in each element has been found to reveal more effectively both the direction and relative intensity of disturbance propagation effects present.

Behaviour of the DRMA when applied to systems with a range of interactive features is now demonstrated by means of two examples drawn from the literature.

2.3 Dominant disturbance propagation (Example 1)

Both Friedly [5] and Jensen *et al.* [7] have presented examples of systems which exhibit so called interactive behaviour. Example 1 is Friedly's system which is presented in state space format as

$$\dot{x}(t) = A_c x(t) + B_c u(t), \tag{5}$$

$$A_c = \frac{1}{1.1}\begin{bmatrix} 0.5 & -5 \\ 0.02 & -2 \end{bmatrix}, \qquad B_c = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$$

and additional transport lags of 0.1 are included in the measurement paths. The RGA for this system is given as

$$\Lambda = \begin{bmatrix} 0.909 & 0.091 \\ 0.091 & 0.909 \end{bmatrix}. \tag{6}$$

Dominance of the diagonal elements suggests that pairing of controlled and manipulated variables is satisfactory and that under conditions of perfect control a low level of interaction will result. Friedly, however, investigates the system transient response under real PI control (controller tuning $K_{c1} = 18$, $T_{I1} = 0.3$, $K_{c2} = 46$, $T_{I2} = 0.25$) and, on finding what appears to be a significant interaction between the loops, proceeds to formulate a criterion to overcome the apparent shortcoming in the RGA in not having predicted this outcome.

Construction of elements of the DRMA for this system leads to the magnitude/log frequency plots shown in Figure 2. Here the diagonal plots show a uniform magnitude value of near unity across the whole frequency range, the low frequency asymptotes corresponding with diagonal elements in the RGA.

This re-emphasises the fact that no interaction is likely at any frequency under the controller tuning in use. The off-diagonal plots in Figure 2 show a similar trend to each other with a peak near the 10 rad time^{-1} frequency but, most significantly,

δ_{12} the magnitude term related to θ_1 is always of the order of 100 times larger than that related to θ_2.

This indicates that the response in θ_1 is far more sensitive to changes in loop 2 than is the response in θ_2 to changes in loop 1. Reaction from loop 1 to loop 2 is negligible and the system is thus not interactive but is in effect the reverse of the disturbance propagation form shown in Figure 1b. To confirm that this is the case consider Figure 3 which shows the step responses in θ_1 and θ_2 produced by Friedly's system for unit step changes in the opposite set points θ_{2sp} and θ_{1sp} respectively. The size of response in θ_2 is in any case negligibly small, but removal of the $G_{21}(s)$ interactor from the system has no significant effect on the response of θ_1 and θ_2.

2.4 True interaction (Example 2)

Example 2, one used by Witcher and McAvoy [14], has been included to demonstrate the behaviour of the proposed DRMA plots for the case of a truly interactive system. Its transfer function from the dynamics is

$$G_p(s) = e^{-s} \begin{bmatrix} \frac{2}{10s+1} & \frac{0.5}{s+1} \\ \frac{0.5}{s+1} & \frac{2}{10s+1} \end{bmatrix}. \tag{7}$$

As can be seen, the structure is completely symmetrical. It is significant to note that the authors are emphatic that their system is a purely synthetic one and they state some doubts about the possibility of its physical realizability. With direct control loop pairing (θ_1 with u_1 etc.) this gives rise to the RGA

$$\Lambda = \begin{bmatrix} 1.06 & -0.06 \\ -0.06 & 1.06 \end{bmatrix} \tag{8}$$

indicating correct pairing and a low level of interaction under conditions of perfect control.

However, with the introduction of real PI controllers (both tuned to $K_c = 1.5$, $T_I = 16.7$) a strong interaction appears in the transient response. The DRMA construction results in Figure 4. The diagonal elements of Λ this time show quite considerable deviations from unity across a broad range of frequencies. Resonances are associated with the deadtime feature. Near the individual loop resonant frequencies the magnitude deviates significantly from unity (*i.e.*, δ_{11} is 3.0 at the loop break frequency of 0.5 rad time^{-1}) indicating a strong interaction in that region of the frequency spectrum. As for the off-diagonal elements, these are identical in from across the whole frequency spectrum indicating that the size of disturbance propagation effects between the two loops (and hence the size of transient deviation in θ_1 and θ_2 following step changes) are the same in both directions, as one might expect in such a symmetric system. Thus the system can be considered to exhibit truly interactive behaviour (*i.e.*, the disturbance propagation is equally balanced in both directions) and this is confirmed in Figure 5, where the set point step responses for the system are presented. Removal of the interactor $G_{12}(s)$ has a significant effect on the responses of both θ_1 and θ_2. Furthermore, the stability (*i.e.*, the level of damping) of those responses is reduced when $G_{12}(s)$ is active — again a sure sign of the presence of the true interaction.

3 Discussion and conclusion

The DRMA approach described here allows a distinction to be drawn between cases of disturbance propagation and cases of true interaction between control loops. An assessment of both the direction and intensity of disturbance propagation between loops is made possible on the basis of frequency response analysis of transfer function models. Further work is currently directed toward extension of the approach outlined here to allow examination of a wider range of proven interactive systems for the strength and importance of disturbance propagational features.

Whilst it is easy on a theoretical basis to synthesise dynamic systems that exhibit strong interaction, the realisation of such behaviour in real world process systems is more difficult and may only arise relatively rarely in practice. The feature of disturbance propagation is by comparison commonplace and where this becomes troublesome, it seems probable that compensation by means of only one-way decoupling will achieve marked improvements. For example, dual composition control in distillation has long been recognised as a challenging interactive control problem. The fact, highlighted by Fagervik *et al.* [4], that considerable improvement in column control quality can be found on the basis of one-way decoupling only, lends support to the conclusion that cases of the dominating disturbance propagation highlighted in this paper may not be that uncommon.

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