5–10 July 2004, Antalya, Turkey — Dynamical Systems and Applications, Proceedings, pp. 193–194

The Modified Quasi-Reversibility Method for an Abstract Ill-Posed Ultraparabolic Problem

N. Boussetila & F. Rebbani
Applied Math Lab, University Badji Mokhtar Annaba
P.O.Box. 12, Annaba 23000, Algeria
E-mail: naboussetila@yahoo.fr; rebbani@wissal.dz

Abstract

A method of solution of ill-posed evolution problems in the strip $[0,T_1] \times [0,T_2]$ is proposed. The method employs a quasi-reversibility approach and is based on the Yosida approximations. For this method, we give a new convergence result and the convergence rate can also be estimated under a priori regularity assumptions on the problem data.

AMS Subject Classification: 35R20, 35R25, 47A52.
Key words: Ultraparabolic equations, Ill-posed problems, Quasi-reversibility method.

Throughout this paper H will denote a complex Hilbert space, endowed with the inner product $(\cdot,\cdot)_0$ and the norm $|\cdot|_0$, $\mathcal{L}(H)$ the Banach algebra of linear bounded operators on H.

Let $D = (0,T_1)\times(0,T_2)$ be a bounded rectangle in the plane $\mathbb{R}^2$ with coordinates $(t,s) \in D$. We consider the initial value problem:

$$\begin{cases} \partial_t u(t,s) + \sigma\, \partial_s u(t,s) + Au(t,s) = 0, \quad (t,s) \in D, \\ u(t,0) = \varphi(t), \quad t \in [0,T_1], \\ u(0,s) = \psi(s), \quad s \in [0,T_2], \end{cases} \qquad (I.V.P.)$$

where u and f are H-valued functions on D, φ (resp. ψ) is an H-valued function on $[0,T_1]$ (resp. $[0,T_2]$) and $\varphi(0) = \psi(0)$; σ is a positive real parameter; A is a positive $(A > 0)$, self-adjoint, unbounded operator on H.

For $(\zeta,\chi) \in L_2((0,T_1);H) \times L_2((0,T_2);H)$ we introduce the functional:

$$\mathcal{F}(\varphi,\psi) = \int_0^{T_1} |u(t,T_2) - \zeta(t)|_0^2\, dt + \int_0^{T_2} |u(T_1,s) - \chi(s)|_0^2\, ds, \qquad (Control)$$

and consider the following minimization problem: find (φ, ψ) such that

$$\mathcal{F}(\varphi, \psi) \leq e \quad (e \approx 0), \tag{M.P.}$$

where $u(t,s) = u(t,s;\varphi,\psi)$ is the solution of problem (I.V.P.).

An obvious solution to the problem (M.P.) is to choose φ, ψ such that $\mathcal{F}(\varphi, \psi) = 0$, *i.e.*, $u(t, T_2) = \zeta(t)$, $u(T_1, s) = \chi(s)$. Hence, in the rectangle D, we consider u as a solution of the following final value problem:

$$\begin{cases} \partial_t v(t,s) + \sigma\, \partial_s v(t,s) + Av(t,s) = 0, \quad (t,s) \in D, \\ v(t, T_2) = \zeta(t), \quad t \in [0, T_1], \\ v(T_1, s) = \chi(s), \quad s \in [0, T_2], \end{cases} \tag{F.V.P.}$$

and we take $v(t,0) = \varphi(t)$, $v(0,s) = \psi(s)$.

Such problems are not well-posed in Hadamard's sense. Even under the best conditions when a solution exists it does not depend continuously on the data.

In this paper, we propose a modified Q.R.-method based on the Yosida approximation ($A_\varepsilon = A(I + \varepsilon A)^{-1}$, $\varepsilon > 0$) as a perturbation. The advantage of this new approach resides in the fact that the perturbation used in our analysis is bounded, which gives a well-posedness in the forward and backward direction for the perturbed problem, the second advantage is that, the Yosida approximation is the best operator approximation which produces a best and significant approximate solution.

The present mathematical model may be interpreted as a generalization of the corresponding well-known model for backward heat conduction problem (B.H.C.) with two time variables, and other modeling as the transport of a contaminant plume through an underground aquifer. Another motivation comes from the fact that the case of inverse multi-time problems does not seem have been widely investigated. This case is caused not only by theoretical interest, but also by practical necessity. Consequently this paper wants to give a contribution in this new field.