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The Uniform Stabilization of the Nonuniform Timoshenko Beam

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Abstract

In this paper we consider the nonuniform Timoshenko beam, we prove that the model can be stabilized by one internal control force .

1 Introduction and results

The equations of motion of a Timoshenko beam are

$$\begin{cases} \alpha w_{tt} = (\beta(\varphi + w_x))_x, \\ \gamma\varphi_{tt}(x,t) = (\delta\varphi_x)_x - \beta(\varphi + w_x), \end{cases} \quad \text{on } (0,1)\times\mathbb{R}^+. \tag{1}$$

The function w is the transverse displacement of the beam and φ is the rotation angle of a filament of the beam. The coefficients α, β, γ and δ are the mass per unit length, the polar moment of inertia of a cross section, Young's modulus of elasticity, the moment of inertia of a cross section and the shear modulus repectively. The natural energy of the beam is

$$\mathcal{E}(t) = \frac{1}{2}\int_0^1 \left\{\alpha \mid w_t \mid^2 + \gamma \mid \varphi_t \mid^2 + \beta \mid \varphi + w_x \mid^2 + \delta \mid \varphi_x \mid^2\right\} dx. \tag{2}$$

The aim of this paper is to study the internal stabilization of this system. We will assume that

$$\alpha, \beta, \gamma \textbf{ and } \delta \textbf{ are positive } \mathbf{C}^1 \textbf{ functions of } x. \tag{3}$$

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We will first prove that it is possible to stabilize uniformly (with respect to the initial data) this beam, by using a unique internal feedback acting only on the rotation angle, namely:

$$\begin{cases} \alpha w_{tt} = (\beta(\varphi + w_x)_x \, , \\ \gamma\varphi_{tt}(x,t) = (\delta\varphi_x)_x - \beta(\varphi + w_x) - a(x)\varphi_t \, , \end{cases} \quad \text{on } (0,1)\times\mathbb{R}^+, \tag{4}$$

and we consider two boundary cases

$$w(0,t) = w(1,t) = 0, \qquad \varphi(0,t) = \varphi(1,t) = 0, \tag{5}$$

and

$$w_x(1,t) = 0, \qquad w(0,t) = \varphi(0,t) = \varphi(1,t) = 0, \tag{6}$$

where a **is a positive continuous function of the space variable**. Indeed, we prove the uniform stability holds for system (4), (5) or (4), (6) provided that the wave speeds $\frac{\delta}{\gamma}$ and $\frac{\beta}{\alpha}$ are the same on the whole interval. If the wave speeds are different on the whole interval, we prove the asymptotic stability and the nonuniform stability.

The first analysis for a Timoshenko beam with variable physical parameters seems to be the one of S. W. Taylor [7]. He studied the boundary control of system (1) and considered two situations, one of them being the case where a force f and a torque τ are both applied to the free end, the beam being clamped at the other.

Our method is based on the computation of the essential type of the associated semigroups for the nonuniform stability, thanks to the result of Neves *et al.* [5] if

$$\sqrt{\frac{\beta}{\alpha}} \neq \sqrt{\frac{\delta}{\gamma}} \qquad \text{on } [0,1],$$

and the construction of the Lyapunov function for the uniform stability if

$$\sqrt{\frac{\beta}{\alpha}} = \sqrt{\frac{\delta}{\gamma}} \qquad \text{on } [0,1].$$

Our results is the following:

Theorem 1 *Under the assumption* (3), *assume that* $a \in C([0,1])$ *and*

$$a \geq a_0 > 0 \qquad \textit{on } (0,1).$$

Then:

- *If* $\dfrac{\beta}{\alpha} \neq \dfrac{\delta}{\gamma}$ *on* $(0,1)$*, then the systems* (4), (5) *or* (4), (6) *are asymptotically stable but not uniformly stable.*
- *If* $\dfrac{\beta}{\alpha} = \dfrac{\delta}{\gamma}$ *on* $(0,1)$*, then the systems* (4), (5) *or* (4), (6) *are uniformly stable.*

As an immediate consequence of this result, we get

Corollary 2 ([6]) *If the physical parameters are constant, then the system is uniformly stable if and only if* $\dfrac{\beta}{\alpha} = \dfrac{\delta}{\gamma}$.

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