Answer all the questions

Show all of your work

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1. Let \( x : 3t^3 + 4t^2 \leq 12t + 3 \), and \( y : 2t^2 + 4t \geq 5 \) represents a parametric curve, then find \( t \) at which the curve has horizontal tangent.

2. Find equation of the tangent plane to the surface \( z : 2x^2 + y^2 \) that is parallel to the plane \( x + 2y \leq z + 3 \leq 0 \).

3. Determine whether the limit exist or not (show all details)

\[
\lim_{\substack{y, z \to 0, 0; \\ x \to 0}} \frac{xy^2}{x^2 + y^4}
\]

4. Sketch the two polar curves \( r : 2\cos 3\theta \), and \( r : 1 \) and give the angles of all points of intersection.

5. Find the distance between the two skew lines

\[
L_1 : x : 2t + 1, \ y : t \leq 2, \ z : 3t + 2 \\
L_2 : x : 3t + 3, \ y : 2t + 2, \ z : t + 2
\]

6. Let \( z : \int \sqrt{2}x + 3y + \cos \theta y \), where \( f \) is a differentiable function of \( x \) and \( y \). Find \( \frac{\partial^2 z}{\partial x \partial y} \).

7. Sketch the polar region \( r : 1 \leq 2 \sin \theta \) and find the area inside the smaller loop.

8. Find the point of intersection between the line \( x : t \leq 4 \), \( y : 2t + 1 \), \( z : 2t + 2 \) and the plane \( 2x \geq y + 3z + 3 \leq 0 \).

9. Find the volume of the solid in the first octant which is between the surfaces \( z : 4 \geq x^2 + y^2 \), and \( z : 3x^2 + 3y^2 \).

10. Find equation of the plane that is perpendicular to the plane \( 2x \geq 2y + z + 5 \geq 0 \) and containing the line \( x : t + 1 \), \( y : 2t + 1 \), \( z : 2t + 3 \).

11. Find the maximum and minimum of the function \( \int \sqrt{2}x + y \), where \( f \) is a differentiable function of \( x \) and \( y \) over the closed rectangle with vertices \( (0, 0), (0, 3), (4, 0), (4, 3) \).

12. Use polar double integral to evaluate the integral \( \iint_Q \sqrt{x^2 + y^2} \, dA \), where \( Q \) is the region in the first quadrant inside the circle \( x^2 + y^2 \leq 1 \).

13. Use double integral to find the area bounded by the curves \( y : x^2 \), \( y : 2x \leq 1 \), and \( y \) \( axis \).

14. Let \( \int \sqrt{2}x + y \), then find the maximum directional derivative of \( \int \sqrt{2}x + y \) at \( (3, 2, 1) \).

15. Set up the equivalent spherical triple integral that is equivalent to triple integral

\[
\iiint_0^1 \iiint_0^{\pi/2} \iiint_0^{\pi/2} \sqrt{x^2 + y^2 + z^2} \, dz \, dy \, dx
\]

16. For each of the following give a short answer in the assigned space:

- a. A normal vector to the plane \( 2x \geq y + 5 \leq 0 \) is equal to

- b. The graph of the polar equation \( r : 4\cos \theta \) has an equivalent rectangular equation equals to
c. The graph of the level surface for \( f(x, y, z) = 3x^2 + y^2 - z^2 \) that passes through the point \( P(1, 2, 1) \) is called ________________________

d. The cylindrical surface \( z = r \) has an equivalent rectangular equation to be ________________________

e. Let \( f(x, y) = x \sin(2y^2) \), then find \( f_x : \)

\[ f_x = \]

g. The line \( x = 2t + 1 \), \( y = 3t + 4 \), \( z = 2t + 2 \) has a parallel vector equals to ________________________

h. The two vectors \( v : \vec{a}, a, 1 \times \), and \( u : \vec{a}, 1, ?1 \times \) are perpendicular if \( a \) equals to ________________________

i. \[ \int_0^1 6x^2 \, dy \, dx \] is equal to ________________________

j. A unit vector parallel to \( b : \vec{a}, 2, ?1 \times \) is ________________________
Answer all the questions
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All the questions have equal mark

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1. Find the area of the triangle with vertices \( P(1,0,1), Q(2,1,0), \) and \( R(0,1,2) \)
2. Find the angles at which the the polar curve \( r = 1 - \cos \theta \) has horizontal tangent line
3. Determine whether the two lines are parallel, intersecting, or skew; \( L_1 : x = 2 + 2t \quad y = 2 + t \quad z = 2t \), \( L_2 : x = 2t \quad y = 1 - 2t \quad z = 2 + t \)
4. Sketch the polar curves \( r = 2 \cos 2\theta \), and \( r = 1 \), and find the polar coordinates of all the points of intersection.
5. Find the distance from the point \( P(0,2,1) \) to the line \( L : x = 2 + 2t \quad y = 2 + 2t \quad z = 2t \).
6. Find the area inside \( r = 1 + \sin \theta \) and outside \( r = 1 \)
7. Let \( u \) and \( v \) be vectors in 3-space, show that \( u \cdot v = \frac{1}{4} u \cdot u + v \cdot v - \frac{1}{4} u \cdot v \cdot v \)
8. For each of the following give a short answer in the assigned space:

   a. Find \( a \) so that the vector \( u = \hat{\theta}, 2a, a \times \) is perpendicular to the vector \( v = \hat{\theta}, 2, \hat{\theta} \times \)

   b. Let \( P(1, \sqrt{3}) \) with rectangular coordinates. Find equivalent polar coordinates

   c. Find parametric equations of the line through \( P(0,2,1) \) and parallel to the vector \( v = \hat{\theta} 2, 0, 3 \times \)

   d. Let \( a = \hat{\theta} 1, 0, 1 \times \), and \( b = \hat{\theta} 1, 0 \times \) Find \( a \times b \)
e. Give the center and the radius of the sphere with the equation
\[ x^2 + y^2 + z^2 + 4x - 8y + 2z + 5 = 0 \]

King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 201 Sem II 2004-2005
Second Major Exam Wed 27 / 4 / 2005 Time 1 \frac{1}{2} 

Name: __________________________ I.D.#: __________________________ Serial #: ___

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1. Use local linear approximation to estimate the volume of a cylindrical tank with radius \( r = 3.02 \text{ m} \), and height \( h = 4.96 \text{ m} \), where \( V = \pi r^2h \)
2. Find equation of the plane that contains the point \( P(2, 1, 1) \) and the line \( x = 2 + t, y = 1 + 2t, z = 2t \)
3. Determine whether the limit exist or not (show all details)
\[
\lim_{(x,y,z) \to (0,0,0)} \frac{xy + xz + yz}{x^2 + y^2 + z^2}
\]
4. Sketch the region enclosed by the paraboloid \( 2z = 3 - x^2 - y^2 \), and the cone \( x^2 + y^2 - z^2 = 0 \), and describe their curve(s) of intersection.
5. Find the distance between the point \( P(2, 1, 1) \), and the plane determined by the points \( Q(0, 2, 1), R(1, 1, 0), S(0, 2, 1) \)
6. Let \( z = f(x, y) + xy \), where \( f \) is a differentiable function of \( x \) and \( y \). Find \( \frac{\partial z}{\partial x} + 3 \frac{\partial z}{\partial y} \)
7. For each of the following give a short answer in the assigned space:
a. The normal vector to the plane \( x + 2y + 3z + 1 = 0 \) is equal to

_____________________

b. The graph of the spherical equation \( r = 4 \) has an equivalent rectangular equation equals to

_____________________

c. The level surface for \( f(x, y) = 3x^2 + 2y^2 \) that passes through the point \( P(1, 2) \) is

_____________________

d. The quadric surface \( x^2 + 9y^2 - 4z^2 + 8z + 3 = 0 \) is called

_____________________

e. Let \( f(x, y, z) = \sin(x^2 + 2y^2 + 4z^2) \), then find \( \frac{f(3, 2, 1)}{z} \)

_____________________

f. Let the point \( P(\sqrt{2}, \frac{Z}{4}, 3) \) with cylindrical coordinates then the equivalent rectangular coordinates are ______________

and spherical coordinates are_____________________

g. If \( z = f(x, y) = 3x^2 + xy \), and \( x = 3t \), \( y = 2t \), then find \( \frac{dz}{dt} \) at \( t = 1 \)

_____________________

Math 201-10 Quize #1B Sem 042
Name:_______________________ I.D.#____________ Serial #_____
Q1: Sketch the graph of the polar curve \( r : 3 + 3 \cos \theta \), show the angles where the graph passes through the pole.

Q2: Find the slope of the tangent line to the parametric curve \( x : t^2 \), \( y : \sin t \) at \( t : \frac{\pi}{3} \).

Q3: Find the angles where the graph \( r : \cos \theta \), \( 0 \leq \theta \leq \pi \) has horizontal tangent.

Math 201-10 Quize #2B Sem 042
Name:_______________________ I.D.#____________ Serial #_____

Q1: Find the area inside \( r : 2 \cos \theta \) and outside \( r : 1 \).

Q2: Find the equation of the sphere centered at \( \bar{y}2, 1, 2 \) and passes through the origin.

Q3: Find the vectors \( u \) and \( v \) in 2-space where \( 2u + v : 2i + 3j \) and \( u - 3v : 2i + 2j \)

Math 201-10 Quize #2A Sem 042
Name:_______________________ I.D.#____________ Serial #_____

Q1: Find the area inside \( r : 2 \sin \theta \) and outside \( r : 1 \).

Q2: Find the equation of the sphere centered at \( \bar{y}1, 2, -2 \) and passes through the origin.

Q3: Find the vectors \( u \) and \( v \) in 2-space where \( 3u + 2v : i - 2j \) and \( u + 3v : 2i + j \)

Math 201-10 Quize #3A Sem 042
Name:_______________________ I.D.#____________ Serial #_____
Q1: Find equation of the plane that contains the line
\[ x = 2 + t, \quad y = 1 + 2t, \quad z = 2t \]
and the point \( P(1, 0, -1) \).

Q2: Identify and sketch the surface \( 9x^2 + 4y^2 + 9z^2 + 36 = 0 \).

Q3: Find distance between the line \( L_1 : x = 1 + 2t, \quad y = 1 - 2t, \quad z = 2t \)
and the plane \( x + 4y + 2z + 2 = 0 \).

Q4: Find equivalent spherical coordinates of the point with rectangular coordinates \( P(2, 2, 0) \).

Q1: \( \lim_{(x, y, z) \to (0, 0, 0)} \frac{\sin\sqrt{x^2 + y^2 + z^2}}{x^2 + y^2 + z^2} : \)

Q2: \( \lim_{(x, y) \to (1, 0)} \frac{\tan\sqrt{x^2 + y^2 + 1}}{x^2 + y^2 + 1} : \)

Q3: Find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) by using implicit differentiation: \( y^2z + \cos\sqrt{xyz} = 0 \).

1. Find all the relative extrema and saddle points (if exist) of the function \( f(x, y) : 3x^2 + 2y^3 + y^2 \).
2. Find the absolute extremum of \( f(x, y) : 2x + 2y^2 \) over the rectangular region with vertices \( \{0, 0\}, \{1, 0\}, \{0, 1\}, \{1, 1\} \).
3. Evaluate the double integral \( \iint_R (x^2y + 2y) \, dA \) where \( R \) is the rectangle \( \{0, 1\} \times \{0, 2\} \).
1. Find all the relative extrema and saddle points (if exist) of the function
   \[ f(x, y) = 3y^2 + 2x^2 x^2 \]
2. Find the absolute extremum of \( f(x, y) = 2y x^2 \) over the rectangular region with vertexes \( \{0, 0\}, \{1, 0\}, \{0, 1\}, \{1, 1\}\).
3. Evaluate the double integral \( \iint_R xy + 4x \, dA \) where \( R \) is the rectangle \( \{0, 2\} \times \{0, 1\} \).

1. Use polar to evaluate the integral \( \int_0^{\pi/2} \int_0^1 \sqrt{x^2 + y^2} \, dx \, dy \).
2. Find the volume of the solid in the first octant bounded by \( z = 9 - y^2, z = 0, x = 0, \) and \( y = x \).
3. Express the integral as an equivalent integral with the order of integration reversed \( \int_0^1 \int_0^1 \int_0^1 \).

1. Set up the triple integral to find the volume of the solid bounded by the surfaces \( z = x^2 + y^2, \) and \( z = 2 - x^2 - y^2 \).
2. Evaluate \( \iiint_G y \, dV \) where \( G \) is the region bounded by the surfaces \( y = x^2, z + y = 4, \) and \( z = 0 \).
3. Express the integral \( \iiint_0^1 \int_0^1 \int_0^1 f(x, y, z) \, dy \, dz \) as integral in the given order \( \int_0^1 \int_0^1 \int_0^1 \) (set up the new limits).
1. Set up the rectangular triple integral to find the volume of the solid in the first octant bounded by the surfaces \( z = 1 - y^2 \), and \( x = 2 \), \( x = 0 \), \( y = 0 \), and \( z = 0 \).

2. Let \( \iiint G (x^2 + y^2 + z^2) \, dV \) where \( G \) is the region below the sphere \( x^2 + y^2 + z^2 = 2 \) and above the paraboloid \( z = x^2 + y^2 \).
   
   a. Set up the triple integral by using Cylindrical Coordinates.
   
   b. Set up the integral by using Spherical coordinates