Math 201 - 1 Quiz # 4 A Sem 053

Name: Key I.D.#: ________________ Serial #: _____

Q1 Evaluate the limit, if exist, by converting to polar coordinates

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$$, $x = r\cos\theta$, $y = r\sin\theta$

as $(x,y) \to (0,0)$, $r \to 0^+$

$$= \lim_{r\to 0^+} \frac{r^2\cos\theta\sin\theta}{r} = \lim_{r\to 0^+} r\cos\theta\sin\theta = 0$$

regardless of θ,

So limit exist and equals to 0.

Q2 Describe the largest region on which the function $f(x,y,z) = \ln(1-x^2-y^2-z^2)$ is continuous. f is continuous if ln is defined,

So $1-x^2-y^2-z^2 > 0 \Rightarrow$ if $x^2+y^2+z^2=1$ (sphere of radius 1, centered at the origin

The region should be all the points inside the sphere.

Q3 Let $f(x,y,z) = x^2+y^2-z^2$. Find an equation of the level surface that passes through the point $(-1,1,-2)$, $f(-1,1,-2) = 1+1-4 = -2$

So $x^2+y^2-z^2=-2$ is the level surface,

it is $-\frac{x^2}{2} - \frac{y^2}{2} + \frac{z^2}{2} = 1$ (Hyperboloid of Two sheets)

Math 201 - 1 Quiz # 4 B Sem 053

Name: Key I.D.#:________ Serial #:_____

Q1 Evaluate the limit, if exist, by converting to polar coordinates

$$\lim_{(x,y)\to(0,0)} \frac{\sin\sqrt{x^2+y^2}}{x^2+y^2}$$

$x = r\cos\theta$, $y = r\sin\theta$

$x^2+y^2 = r^2$, as $(x,y)\to(0,0)$, $r\to 0^+$

$$\lim_{r\to 0^+} \frac{\sin r}{r^2} = \lim_{r\to 0^+} \frac{1}{r}\cdot \frac{\sin r}{r} \to 1 = \infty$$

Q2 Describe the largest region on which the function $f(x,y,z) = \frac{-1}{4-x^2-y^2-z^2}$ is continuous.

f is continuous if $4-x^2-y^2-z^2 \neq 0$

$x^2+y^2+z^2 = 4$ (sphere centered at the origin with radius 2)

So the region at which f is continuous is all 3-space except the sphere.

Q3 Let $f(x,y,z) = x^2-y^2-z^2$. Find an equation of the level surface that passes through the point $(-2,1,-1)$

$f(-2,1,-1) = 4-1-1 = 2$

$$x^2-y^2-z^2 = 2 \Leftrightarrow \frac{x^2}{2} - \frac{y^2}{2} - \frac{z^2}{2} = 1$$

Hyperboloid of Two sheets

Math 201 - H Quiz # 4 Sem 053

Name:________________________ I.D.#:________________ Serial #:_____

Q1 Evaluate the limit, if exist, by converting to ~~polar~~ spherical coordinates

$$\cancel{\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}} \qquad \lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2}$$

$x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho \cos\phi$

as $(x,y,z) \to (0,0,0)$, then $\rho \to 0^+$

$$\lim_{\rho\to 0} \frac{\rho^3 \sin^2\phi \cos\phi \sin\theta \cos\theta}{\rho^2} = \lim_{\rho\to 0^+} \rho \sin^2\phi \cos\phi \sin\theta \cos\theta$$

$$= 0$$

Q2 Describe the largest region on which the function $f(x,y,z) = \cancel{\ln(1-x^2-y^2-z^2)}$ $\frac{-1}{\sqrt{9-x^2-y^2}}$ is continuous .

~~f is continuous if $1-x^2-y^2-z^2 > 0 \Rightarrow x^2+y^2+z^2 < 1$~~

~~So the region is the interior of the sphere~~

~~Centered at (0,0,0) with radius 1~~

$9-x^2-y^2 > 0 \Rightarrow x^2+y^2 < 9$

So the region is inside the circle $x^2+y^2=9$ but not on the circle.

Q3 Let $f(x,y,z) = -x^2 - y^2 + z^2$. Find an equation of the level surface that passes through the point $(-2,2,-1)$

~~$f(-1,1,-2) = 1+1-4 = -2$~~

~~The equation is: $x^2+y^2-z^2 = -2$~~

~~$-\frac{x^2}{2} - \frac{y^2}{2} + \frac{z^2}{2} = 1$ Hyperboloid of Two sheets~~

$-4-4+1 = -7$

$-x^2-y^2+z^2 = -7$ / $\frac{x^2}{7} + \frac{y^2}{7} - \frac{z^2}{7} = 1$

Hyp. of one sheet.