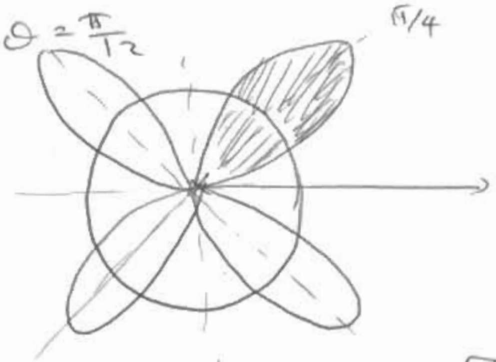
Math 201 - 4 Quiz # 2 Sem 053

Name: Key I.D.#: ________ Serial #: _____

Q1 Find the area inside the polar curve $r = 2\sin 2\theta$ and outside the curve $r = 1$

$\sin 2\theta = \frac{1}{2}$ $\theta = \frac{\pi}{12}$

$2\theta = \frac{\pi}{6} \Rightarrow$



$$8 \cdot \frac{1}{2} \int_{\pi/12}^{\pi/4} \left[(2\sin 2\theta)^2 - 1 \right] d\theta$$

$$= 4 \int_{\pi/12}^{\pi/4} \left[4\sin^2 2\theta - 1 \right] d\theta = 4 \int_{\pi/12}^{\pi/4} \left[2 - 2\cos 4\theta - 1 \right] d\theta = 4 \left[\theta - \frac{1}{2}\sin 4\theta \right]_{\pi/12}^{\pi/4}$$

$$= 4\left(\frac{\pi}{4} - \frac{\pi}{12}\right) - 2\left(0 - \frac{\sqrt{3}}{2}\right) = \frac{2\pi}{3} + \frac{\sqrt{3}}{2}.$$

Q2 Find equation of the sphere that is centered at $C(0,3,-1)$ and contains the center of the sphere $x^2 + y^2 + z^2 - 4x - 4y - 2z + 5 = 0$

$x^2 - 4x + 4 + y^2 - 4y + 4 + z^2 - 2z + 1 = -5 + 4 + 4 + 1$

$(x-2)^2 + (y-2)^2 + (z-1)^2 = 4$ $\quad C_1(2,2,1)$, $r = 2$

$d(C_1, C) = \sqrt{4 + 1 + 4} = 3 =$ radius of the sphere

$\therefore$ The equation is: $x^2 + (y-3)^2 + (z+1)^2 = 9$

Q3 Let the vectors $\mathbf{u} = \langle 1,-2,0 \rangle$, and $\mathbf{v} = \langle 1,2,1 \rangle$, find a vector parallel to $3\mathbf{v} - 2\mathbf{u}$ with magnitude equals to 3.

$3v - 2u = 3\langle 1,2,1 \rangle - 2\langle 1,-2,0 \rangle = \langle 1,10,3 \rangle$

a unit vector $= \dfrac{3v - 2u}{\|3v - 2u\|} = \dfrac{\langle 1,10,3 \rangle}{\sqrt{1+100+9}} = \dfrac{\langle 1,10,3 \rangle}{\sqrt{110}}$

of magnitude 3 $= 3\dfrac{\langle 1,10,3 \rangle}{\sqrt{110}} = \dfrac{\langle 3,30,9 \rangle}{\sqrt{110}}$