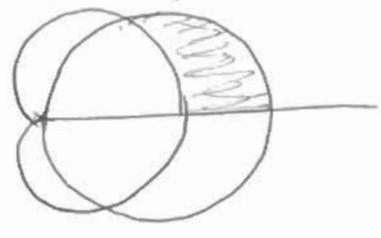
Math 201 - 1 Quiz # 2 A Sem 053

Name: Key I.D.#:________ Serial #:_____

Q1 Find the area inside the polar curve $r = 3\cos\theta$ and outside the curve $r = 1+\cos\theta$

$3\cos\theta = 1+\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$$A = 2\cdot\frac{1}{2}\int_0^{\pi/3}\left[9\cos^2\theta - (1+\cos\theta)^2\right]d\theta$$



$$= \int_0^{\pi/3}\left[9\cos^2\theta - 1 - 2\cos\theta - \cos^2\theta\right]d\theta$$

$$= \int_0^{\pi/3}(4 + 4\cos 2\theta - 2\cos\theta - 1)\,d\theta = \left[3\theta + 2\sin 2\theta - 2\sin\theta\right]_0^{\pi/3}$$

$$= \pi + 2\cdot\frac{\sqrt{3}}{2} - 2\frac{\sqrt{3}}{2} = \pi$$

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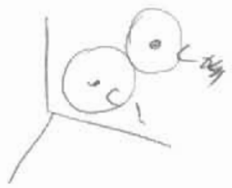
Q2 Find equation of the sphere that is centered at $C(-1,1,1)$ and tangent from outside to the sphere $x^2+y^2+z^2-2x-4y-6z+5=0$

$(x-1)^2+(y-2)^2+(z-3)^2 = 9 \qquad C_1(1,2,3), \quad \underline{r=3}$.

$d(C, C_1) = \sqrt{4+1+4} = 3$



since $r=3$ & $d(C,C_1)=3 \Rightarrow$

and since $r_1 + r = 3 \Rightarrow r_1 = 0$

So the sphere is with radius 0, i.e. it is a point at $(-1,1,1)$.

Q3 Let the vectors $\mathbf{u} = \langle 2,-1,1\rangle$, and $\mathbf{v} = \langle 0,1,-1\rangle$, find a unit vector parallel to the curve $2\mathbf{u}-3\mathbf{v}$

$2u - 3V = 2\langle 2,-1,1\rangle - 3\langle 0,1,-1\rangle$

$= \langle 4,-5,5\rangle$.

a unit vector $\dfrac{2u-3V}{\|2u-3V\|} = \dfrac{\langle 4,-5,5\rangle}{\sqrt{16+25+25}} = \dfrac{\langle 1,-5,5\rangle}{\sqrt{66}}$