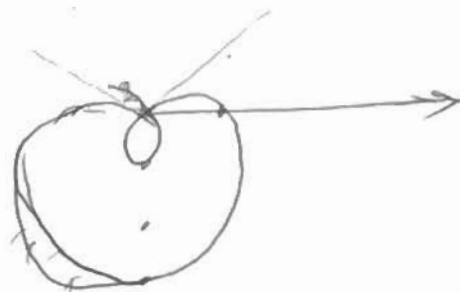
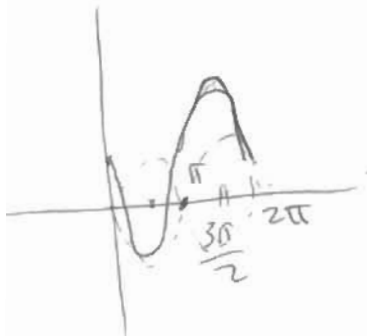


Key

Q1 Sketch the graph of the polar curve  $r = 1 - 2\sin\theta$ 

$$r=0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Q2 Find the equation of the tangent line to the curve  $x = 2t - 1$ ,  $y = t^2 + 2t + 1$  at $t = 2$ 

$$x' = 2, \quad y' = 2t + 2, \quad \frac{dy}{dx} = \frac{2t+2}{2} = t+1 \Big|_{t=2} = 3$$

$$x = 3, \quad y = 9$$

$$y - 9 = 3(x - 3) = 3x - 9$$

$$y = 3x$$

Q3 Find all points at which the polar curve  $r = 3\cos\theta$  has a vertical tangent.

$$\frac{dr}{d\theta} = -3\sin\theta, \quad \text{vertical tangent if } \frac{dx}{d\theta} = 0 \text{ and } \frac{dy}{d\theta} \neq 0$$

$$x = r\cos\theta = 3\cos^2\theta, \quad \frac{dx}{d\theta} = -6\cos\theta\sin\theta$$

$$\frac{dx}{d\theta} = 0 \Rightarrow -6\cos\theta\sin\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin\theta = 0 \Rightarrow \theta = 0, \pi, 2\pi$$

$$\frac{dy}{d\theta} = 3\cos^2\theta - 3\sin^2\theta$$

$$\frac{dy}{d\theta} \Big|_{\theta=0} \neq 0, \quad \frac{dy}{d\theta} \Big|_{\theta=2\pi} \neq 0, \quad \frac{dy}{d\theta} \Big|_{\theta=\frac{\pi}{2}} \neq 0, \quad \frac{dy}{d\theta} \Big|_{\theta=\frac{3\pi}{2}} \neq 0$$

The points are  $P(3, 0)$ ,  $P_2(0, \frac{\pi}{2})$  since  $(-3, \pi) = (3, 0) + (0, \frac{\pi}{2}) = (0, \frac{3\pi}{2})$