Math 201-13 Quize #7A Sem 041

Name: Key I.D.#__________ Serial #_____

Q1: Evaluate the double integral $\iint_R 6(x^2+y^2)\,dA$, where R is the rectangle $\{(x,y) : 1 \le x \le 3, \quad 0 \le y \le 2\}$

$$\int_1^3 \int_0^2 6(x^2+y^2)\,dy\,dx = \int_1^3 \left[6\left(x^2 y + \frac{y^3}{3}\right)\right]_0^2 dx$$

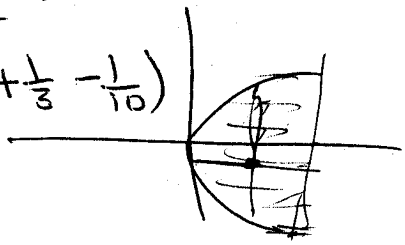
$$= \int_1^3 [12x^2 + 16]\,dx = 12\frac{x^3}{3} + 16x\Big]_1^3 = 4(26) + 16\cdot 2 = 136$$

Q2: Use double integral to find the volume of the solid enclosed by $x = y^2$, $z = 0$, and $z + x = 1$

$$V = \iint_R f(x,y)\,dA = \int_{-1}^1 \int_{y^2}^1 (1-x)\,dx\,dy$$

$$= \int_{-1}^1 \left(x - \frac{x^2}{2}\right]_{y^2}^1 dy = \int_{-1}^1 \left[1 - y^2 - \frac{1}{2} + \frac{y^4}{2}\right] dy$$

$$= \left[\frac{y}{2} - \frac{y^3}{3} + \frac{y^5}{10}\right]_{-1}^1 = \frac{1}{2} - \frac{1}{3} + \frac{1}{10} - \left(\frac{-1}{2} + \frac{1}{3} - \frac{1}{10}\right)$$



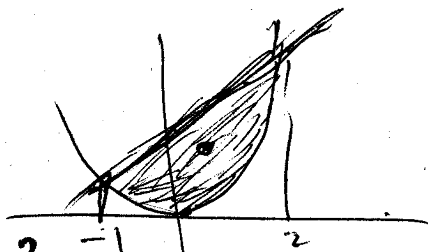
$$= 1 - \frac{2}{3} + \frac{\cancel{2}\,1}{\cancel{10}\,5} = \frac{15 - 10 + 3}{15} = \frac{8}{15}$$

Q3: Use double intgral to find the area of the region enclosed by the curves $y = x^2$, and $y = x + 2$

$x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0 \Rightarrow$

$x = 2 \text{ or } x = -1$



$$A = \iint_R 1\,dA = \int_{-1}^2 \int_{x^2}^{x+2} dy\,dx = \int_{-1}^2 (x + 2 - x^2)\,dx = \frac{x^2}{2} + 2x - \frac{x^3}{3}\Big]_{-1}^2$$

$$= \frac{4}{2} - \frac{1}{2} + 4 + 2 - \frac{8}{3} - \frac{1}{3} = \frac{3}{2} + 6 - 3 = \frac{9}{2}$$

Math 201-16 Quize #7A Sem 041

Name: Key I.D.#__________ Serial #_____

Q1: Evaluate the double integral $\iint_R 6(x^2+y)dA$, where R is the rectangle

$\{(x,y) : 1 \le x \le 2, \quad 0 \le y \le 3\}$

$$\int_1^2\int_0^3 6(x^2+y)\,dy\,dx = \int_1^2 \left[6x^2y + 6\frac{y^2}{2}\right]_0^3 dx$$

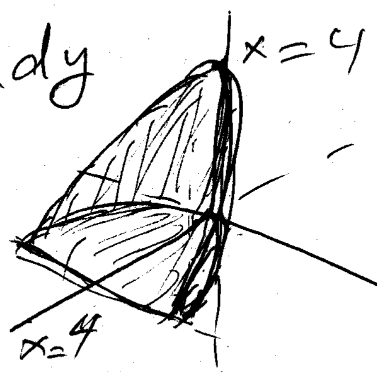
$$= \int_1^2 [18x^2+27]\,dx = \left[\frac{18x^3}{3}+27x\right]_1^2 = 48+54-33$$

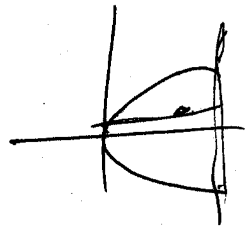
$$= 69$$

Q2: Use double integral to find the volume of the solid enclosed by

$x = y^2$, $z = 0$, and $z = 4 - x$

$$V = \iint_R (4-x)\,dA = \int_{-2}^2\int_{y^2}^4 (4-x)\,dx\,dy$$





$$= \int_{-2}^2 \left(4x - \frac{x^2}{2}\right]_{y^2}^4 dy$$

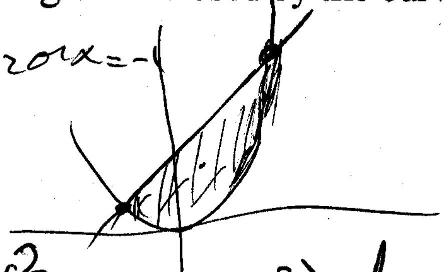
$$= \int_{-2}^2 \left[16-8-4y^2+\frac{y^4}{2}\right]dy = 2\left[8y - 4\frac{y^3}{3}+\frac{y^5}{10}\right]^2$$

$$= 2\left[16-\frac{32}{3}+\frac{16}{5}\right] = 2\,\frac{240-160+48}{15} = \frac{256}{15}$$

Q3: Use double intgral to find the area of the region enclosed by the curves

$y = x^2$, and $y = x + 2$

$x^2 = x+2 \Rightarrow x^2-x-2=0 \Rightarrow x=2$ or $x=-1$



Type I region:

$$A = \iint_R 1\,dA = \int_{-1}^2\int_{x^2}^{x+2} dy\,dx = \int_{-1}^2 (x+2-x^2)\,dx = \left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^2$$

$$= \frac{4}{2}+4-\frac{8}{3}-\left(\frac{1}{2}-2+\frac{1}{3}\right) = 8-\frac{1}{2}-3 = \frac{9}{2}$$

Math 201-13 Quize #7B Sem 041

Name: Key I.D.#__________ Serial #_____

Q1: Evaluate the double integral $\iint_R 3(x^2+y^2)dA$, where R is the rectangle

$\{(x,y) : 0 \le x \le 3, \quad 1 \le y \le 3\}$

$$I = \int_0^3 \int_1^3 3(x^2+y^2)\,dy\,dx = \int_0^3 \left[3x^2y + \frac{3y^3}{3}\right]_1^3 dx$$

$$= \int_0^3 [6x^2+26]\,dx = \frac{6x^3}{3} + 26x\Big]_0^3 = 54 + 78$$

$$= 132$$

Q2: Use double integral to find the volume of the solid enclosed by

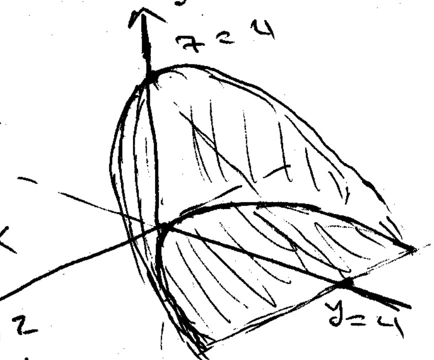
$y = x^2$, $z = 0$, and $z = 4 - y$

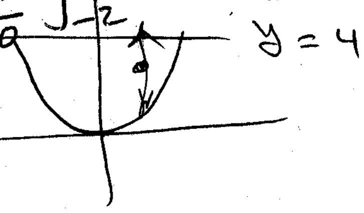
$$V = \iint_R f(x,y)\,dA = \iint_R (4-y)\,dA$$

$$= \int_{-2}^2 \int_{x^2}^4 (4-y)\,dy\,dx = \int_{-2}^2 \left[4y - \frac{y^2}{2}\right]_{x^2}^4 dx$$

$$= \int_{-2}^2 \left[16 - 4x^2 - 8 + \frac{x^4}{2}\right] dx = \left[8x - \frac{4x^3}{3} + \frac{x^5}{10}\right]_{-2}^2$$

$$= \left[32 - \frac{4}{3}(8+8) + \frac{1}{10}(32+32)\right] = \frac{256}{15}$$





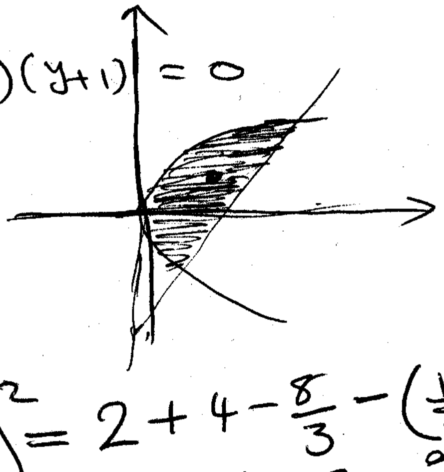
Q3: Use double intgral to find the area of the region enclosed by the curves

$x = y^2$, and $y = x - 2$ $\Rightarrow x = y + 2$

$y^2 = y + 2 \Rightarrow y^2 - y - 2 = 0 \Rightarrow (y-2)(y+1) = 0$

So $y = 2$ or $y = -1$

Type II region:



$$A = \iint_R 1\,dA = \int_{-1}^2 \int_{y^2}^{y+2} 1\,dx\,dy$$

$$= \int_{-1}^2 (y+2-y^2)\,dy = \frac{y^2}{2} + 2y - \frac{y^3}{3}\Big]_{-1}^2 = 2 + 4 - \frac{8}{3} - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 8 - \frac{1}{2} - 3 = \frac{9}{2}$$

Math 201-16 Quize #7B Sem 041

Name: Key I.D.#__________ Serial #_____

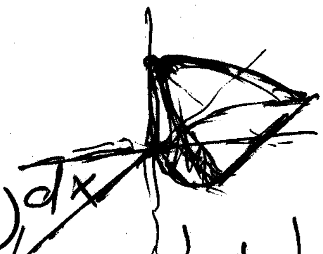
Q1: Evaluate the double integral $\iint_R 6(x+y^2)dA$, where R is the rectangle $\{(x,y) : 0 \le x \le 2, \quad 1 \le y \le 3\}$

$$\int_0^2 \int_1^3 (6x+6y^2)\,dy\,dx = \int_0^2 \left[6xy + 6\frac{y^3}{3}\right]_1^3 dx$$

$$= \int_0^2 (12x+52)\,dx = 12\frac{x^2}{2} + 52x \Big]_0^2 = 24+104$$

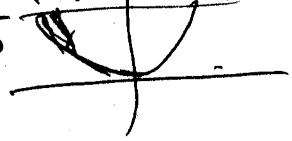
$$= 128$$

Q2: Use double integral to find the volume of the solid enclosed by $y = x^2$, $z = 0$, and $z = 1 - y$



$$V = \iint_R f(x,y)\,dA = \int_{-1}^1 \int_{x^2}^1 (1-y)\,dy\,dx$$

$$= \int_{-1}^1 \left(y - \frac{y^2}{2}\right]_{x^2}^1 dx = \int_{-1}^1 \left(1 - \frac{1}{2} - x^2 + \frac{x^4}{2}\right) dx$$



$$= \frac{x}{2} - \frac{x^3}{3} + \frac{x^5}{10}\Big]_{-1}^1 = \frac{1}{2} - \frac{-1}{2} - \frac{1}{3} + \frac{-1}{3} + \frac{1}{10} + \frac{1}{10}$$

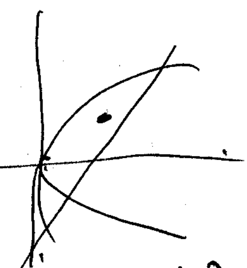
$$= 1 - \frac{2}{3} + \frac{1}{5} = \frac{15-10+3}{15} = \frac{8}{15}$$

Q3: Use double intgral to find the area of the region enclosed by the curves $x = y^2$, and $y = x - 2$

$y^2 = y+2$

$\Rightarrow y=-1$ or $y=2$

Type II region



$$A = \int_{-1}^2 \int_{y^2}^{y+2} dx\,dy = \int_{-1}^2 \left[y+2-y^2\right] dy$$

$$= \frac{y^2}{2} + 2y - \frac{y^3}{3}\Big]_{-1}^2 = \frac{4}{2} + 4 - \frac{8}{3} - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 8 - \frac{1}{2} - 3 = \frac{9}{2}$$