Math 201-16 Quize #5A Sem 041

Name: Key I.D.#________ Serial #_____

Q1: Find the directional derivative of $f(x,y,z) = 4e^{xy}\cos z$ at $P(1,0,\frac{\pi}{4})$ in the direction from P toward the origin.

$f_x = 4ye^{xy}\cos z \Big|_{(1,0,\frac{\pi}{4})} = 0$

$f_y = 4xe^{xy}\cos z \Big|_{(1,0,\frac{\pi}{4})} = 4\cdot\frac{\sqrt{2}}{2} = 2\sqrt{2}$

$f_z = -4e^{xy}\sin z \Big|_{(1,0,\frac{\pi}{4})} = -4\cdot\frac{\sqrt{2}}{2} = -2\sqrt{2}$

$\vec{PO} = \langle -1, 0, -\frac{\pi}{4}\rangle$, $\vec{u} = \frac{\vec{PO}}{\|\vec{PO}\|} = \frac{\langle -1,0,-\frac{\pi}{4}\rangle}{\sqrt{1+\frac{\pi^2}{16}}} = \frac{\langle -4,0,-\pi\rangle}{\sqrt{16+\pi^2}}$

$D_{\vec{u}}f(P) = 0\cdot\frac{-4}{\sqrt{16+\pi^2}} + 2\sqrt{2}\cdot 0 + (-2\sqrt{2})\frac{-\pi}{\sqrt{16+\pi^2}} = \frac{2\sqrt{2}\pi}{\sqrt{16+\pi^2}}$

Q2: Find parametric equations of the normal line to the surface $z = 3x^3y^2 + 2x^2y$, at $P(-1,2,-8)$

$\frac{\partial z}{\partial x} = (9x^2y^2 + 4xy)\Big|_{(-1,2,-8)} = 36 - 8 = 28$

$\frac{\partial z}{\partial y} = 6x^3y + 2x^2\Big|_{(-1,2,8)} = -12 + 2 = -10$

$\vec{v} = \langle 28, -10, -1\rangle$,

The parametric Eq. of the line are:

$x = -1 + 28t$, $y = 2 - 10t$, $z = -8 - t$

Q3: Find parametric equations of the line tangent to the curve of intersection of the cylinders $x^2 + y^2 = 25$, and $y^2 + z^2 = 25$ at the point $S(4,-3,4)$

$F(x,y,z) = x^2 + y^2 - 25$, $G(x,y,z) = y^2 + z^2 - 25$

$\vec{\nabla F} = \langle 2x, 2y, 0\rangle\Big|_{(4,-3,4)} = \langle 8,-6,0\rangle$, $\vec{\nabla G} = \langle 0,-6,8\rangle$

$\vec{v} = \vec{\nabla F}\times\vec{\nabla G} = \begin{vmatrix} i & j & k \\ 8 & -6 & 0 \\ 0 & -6 & 8 \end{vmatrix} = i(-48) - j(64) + k(-48) = -48\vec{i} - 64\vec{j} - 48\vec{k}$

So Para. Eq. are: $x = 4 - 48t$, $y = -3 - 64t$, $z = 4 - 48t$

Math 201-13 Quize #5A Sem 041

Name: Key I.D.#________ Serial #_____

Q1: Find the directional derivative of $f(x,y,z) = 4e^{xy}\cos z$ at $P(0,1,\frac{\pi}{4})$ in the direction from P toward the origin.

$$f_x = 4ye^{xy}\cos z\Big|_{(0,1,\frac{\pi}{4})} = 4\cdot\frac{\sqrt{2}}{2} = 2\sqrt{2}$$

$$f_y = 4xe^{xy}\cos z\Big|_{(0,1,\frac{\pi}{4})} = 0$$

$$f_z = 4e^{xy}(-\sin z)\Big|_{(0,1,\frac{\pi}{4})} = -4\frac{\sqrt{2}}{2} = -2\sqrt{2}$$

$$\vec{\nabla} f = \langle 2\sqrt{2}, 0, -2\sqrt{2}\rangle$$

$$\vec{PO} = \langle 0,-1,\tfrac{-\pi}{4}\rangle \Rightarrow \vec{u} = \frac{\vec{PO}}{\|\vec{PO}\|} = \frac{\langle 0,-1,\frac{-\pi}{4}\rangle}{\sqrt{1+\frac{\pi^2}{16}}} = 4\frac{\langle 0,-1,\frac{-\pi}{4}\rangle}{\sqrt{16+\pi^2}}$$

$$D_{\vec{u}}f(P) = 2\sqrt{2}\cdot 0 + 0\cdot\frac{-4}{\sqrt{16+\pi^2}} + (-2\sqrt{2})\frac{-\pi}{\sqrt{16+\pi^2}} = \frac{2\sqrt{2}\pi}{\sqrt{16+\pi^2}}$$

Q2: Find equation of the tangent plane to ~~to~~ the surface $z = 2x^3y^2 + 3x^2y$, at $P(-1,2,-2)$

$$\frac{\partial z}{\partial x} = 6x^2y^2 + 6xy\Big|_{(-1,2,-2)} = 24-12 = 12$$

$$\frac{\partial z}{\partial y} = 4x^3y + 3x^2\Big|_{(-1,2,-2)} = -8+3 = -5$$

$$\vec{\nabla} f = \langle 12,-8,-1\rangle,$$

So the equation: $12(x+1) - 8(y-2) - (z+2) = 0$

$$12x - 8y - z + 26 = 0$$

Q3: Find parametric equations of the line tangent to the curve of intersection of the cylinders $x^2+y^2=25$, and $y^2+z^2=25$ at the point $S(4,-3,4)$

$F(x,y,z) = x^2+y^2-25$, $G(x,y,z) = y^2+z^2-25$

$$\vec{\nabla} F = \langle 2x,2y,0\rangle\Big|_{(4,-3,4)} = \langle 8,-6,0\rangle,$$

$$\vec{\nabla} G = \langle 0,2y,2z\rangle\Big|_{(4,-3,4)} = \langle 0,-6,8\rangle$$

$$\vec{v} = \vec{\nabla} F\times\vec{\nabla} G = \begin{vmatrix} i & j & k \\ 8 & -6 & 0 \\ 0 & -6 & 8\end{vmatrix} = i(-48) - j(64) + k(-48) = -48\vec{i} - 64\vec{j} - 48\vec{k}$$

param. Eq. are: $x = 4-48t$, $y = -3-64t$, $z = 4-48t$

Math 201-13 Quize #5B Sem 041

Name: Key I.D.#__________ Serial #_____

Q1: Find the directional derivative of $f(x,y,z) = 2e^{xy}\sin z$ at $P(1,0,\frac{\pi}{4})$ in the direction from P toward the origin.

$$f_x = 2y e^{xy}\sin z\Big|_{(1,0,\frac{\pi}{4})} = 0$$

$$f_y = 2x e^{xy}\sin z\Big|_{(1,0,\frac{\pi}{4})} = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$$

$$f_z = 2e^{xy}\cos z\Big|_{(1,0,\frac{\pi}{4})} = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$$

$$\vec{\nabla} f = \langle 0, \sqrt{2}, \sqrt{2}\rangle$$

$$\vec{u} = \frac{\vec{PO}}{\|\vec{PO}\|} = \frac{\langle -1, 0, -\frac{\pi}{4}\rangle}{\sqrt{1+\frac{\pi^2}{16}}} = \left\langle \frac{-4}{\sqrt{1+\pi^2}}, 0, \frac{-\pi}{\sqrt{1+\pi^2}}\right\rangle$$

$$\therefore D_{\vec{u}} f(P) = 0\cdot\frac{-4}{\sqrt{1+\pi^2}} + \sqrt{2}\cdot 0 + \sqrt{2}\cdot\frac{-\pi}{\sqrt{1+\pi^2}} = \frac{-\pi\sqrt{2}}{\sqrt{1+\pi^2}}$$

Q2: Find equation of the tangent plane to the surface $z = 3x^2y^2 + 2xy$, at $P(-1,2,8)$, $z = f(x,y) \Longrightarrow f_x = 6xy^2 + 2y\Big|_{(-1,2,8)} = -24+4 = -20$

$$f_y = 6x^2y + 2x\Big|_{(-1,2,8)} = 12 - 2 = 10$$

$$\vec{\nabla} f(P) = \langle -20, 10, -1\rangle = \vec{n},$$

The eq. is: $-20(x+1) + 10(y-2) - (z-8) = 0$

$$-20x + 10y - z - 32 = 0$$

Q3: Find parametric equations of the line tangent to the curve of intersection of the cylinders $x^2+y^2 = 25$, and $x^2+z^2 = 25$ at the point $S(4,-3,3)$

$F(x,y,z) = x^2+y^2+25$, $G(x,y,z) = x^2+z^2-25$

$F(x,y,z)=0 \to$ is the 1st surface, $G(x,y,z)=0$ is 2nd surf.

$\vec{\nabla} F = \langle 2x, 2y, 0\rangle|_S = \langle 8,-6,0\rangle$, $\vec{\nabla} G = \langle 8,0,6\rangle$

$$\vec{v} = \vec{\nabla} F \times \vec{\nabla} G = \begin{vmatrix} i & j & k \\ 8 & -6 & 0 \\ 8 & 0 & 6 \end{vmatrix} = i(-36) - j(48) + k(48) = -36i - 48j + 48k$$

$\therefore$ Param. eq. are: $x = 4 - 36t$, $y = -3 - 48t$, $z = 3 + 48t$

Math 201-16 Quize #5B Sem 041

Name: Key I.D.#____________ Serial #_____

Q1: Find the directional derivative of $f(x,y,z) = 2e^{xy}\sin z$ at $P(0,1,\frac{\pi}{4})$ in the direction from P toward the origin.

$f_x = 2ye^{xy}\sin z\Big|_{(0,1,\frac{\pi}{4})} = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$

$f_y = 2xe^{xy}\sin z\Big|_{(0,1,\frac{\pi}{4})} = 0$

$f_z = 2e^{xy}\cos z\Big|_{(0,1,\frac{\pi}{4})} = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$

$\Rightarrow \vec{\nabla} f = \langle\sqrt{2}, 0, \sqrt{2}\rangle$

$\vec{PO} = \langle 0, -1, \frac{-\pi}{4}\rangle, \Rightarrow \vec{u} = \frac{\vec{PO}}{\|\vec{PO}\|} = \frac{\langle 0,-1,\frac{-\pi}{4}\rangle}{\sqrt{1+\frac{\pi^2}{16}}} = \frac{\langle 0,-4,-\pi\rangle}{\sqrt{16+\pi^2}}$

$D_{\vec{u}}f(P) = \sqrt{2}\cdot 0 + 0\cdot\frac{-4}{\sqrt{16+\pi^2}} + \sqrt{2}\frac{-\pi}{\sqrt{16+\pi^2}} = \frac{-\sqrt{2}\,\pi}{\sqrt{16+\pi^2}}$

Q2: Find parametric equations of the normal line to the surface $z = 2x^3y^2 - 3x^2y$, at $P(1,2,2)$

$\frac{\partial z}{\partial x} = 6x^2y^2 - 6xy\Big|_{(1,2,2)} = 24 - 12 = 12$

$\frac{\partial z}{\partial y} = 4x^3y - 3x^2\Big|_{(1,2,2)} = 8 - 3 = 5$

$\vec{v} = \langle 12, 5, -1\rangle,$

param. eq. are: $x = 1+12t$

$y = 2+5t$

$z = 2\cancel{+}\,t$

Q3: Find parametric equations of the line tangent to the curve of intersection of the cylinders $x^2+y^2 = 25$, and $x^2+z^2 = 25$ at the point $S(4,-3,3)$

$F(x,y,z) = x^2+y^2-25, \quad G(x,y,z) = x^2+z^2-25$

$\vec{\nabla}F = \langle 2x, 2y, 0\rangle\Big|_S = \langle 8, -6, 0\rangle, \quad \vec{\nabla}G = \langle 8, 0, 6\rangle$

$\vec{v} = \vec{\nabla}F \times \vec{\nabla}G = \begin{vmatrix} i & j & k \\ 8 & -6 & 0 \\ 8 & 0 & 6 \end{vmatrix} = i(-36) - j(48) + k(48)$

$= -36\vec{i} - 48\vec{j} + 48\vec{k}$

: param