Math 201-13 Quize #4A Sem 041

Name: Key I.D.#______ Serial #______

Q1: Describe the graph of the level surfaces for $f(x,y,z) = k$, where $f(x,y,z) = 4x^2 - \frac{y^2}{9} - \frac{z^2}{4}$ and $k = -1,0,4$

$k=-1$: $4x^2 - \frac{y^2}{9} - \frac{z^2}{4} = -1 \Longrightarrow -4x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1$

is Hyperboloid of one sheet

$k=0$: $4x^2 - \frac{y^2}{9} - \frac{z^2}{4} = 0 \rightarrow$ is a cone

$k=4$: $4x^2 - \frac{y^2}{9} - \frac{z^2}{4} = 4 \Longrightarrow x^2 - \frac{y^2}{36} - \frac{z^2}{16} = 1$ is hyperboloid of two sheets.

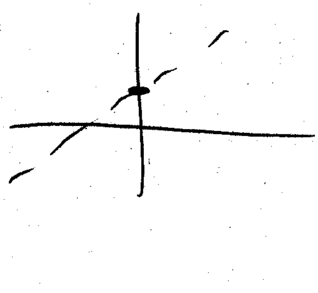
Q2: Give equivalent cylindrical and spherical coordinates of the point P with rectangular coordinates $P(-1, 1, \sqrt{2})$

$x=-1,\ y=1,\ \&\ z=\sqrt{2} \Longrightarrow r = \sqrt{1+1} = \sqrt{2},\ \theta = \tan^{-1}(-1) \Rightarrow \theta = \frac{3\pi}{4}$

$\rho = \sqrt{1+1+2} = 2,\ \tan\phi = \frac{r}{z} = \frac{\sqrt{2}}{\sqrt{2}} = 1 \Rightarrow \phi = \frac{\pi}{4}$

cylindrical coord. $(\sqrt{2}, \frac{3\pi}{4}, \sqrt{2})$

spherical coord. $(2, \frac{3\pi}{4}, \frac{\pi}{4})$



Q3: Determine if the limit exist $\lim_{(x,y)\to(0,1)} \frac{x(y-1)}{x^2+(y-1)^2}$

Along y-axis: $\lim_{(x,y)\to(0,1)} \frac{0}{0+(y-1)^2} = 0$

Along the line $y = x-1 \Longrightarrow \langle 1,1\rangle,\ P_0(0,1)$

$x=t,\ y=t+1$, as $(x,y)\to(0,1) \Longrightarrow t\to 0$

$\lim_{t\to 0} \frac{t(t+1-1)}{t^2+(t+1-1)^2} = \lim_{t\to 0} \frac{t^2}{2t^2} = \frac{1}{2}$

$\therefore$ The limit does not exist.

Math 201-13 Quize #4B Sem 041

Name: Key I.D.#__________ Serial #_____

Q1: Describe the graph of the level surfaces for $f(x,y,z) = k$, where $f(x,y,z) = \frac{x^2}{4} + y^2 - \frac{z^2}{9}$ and $k = -4, 0, 1$

$\frac{x^2}{4} + y^2 - \frac{z^2}{9} = -4 \Rightarrow \frac{-x^2}{16} - \frac{y^2}{4} + \frac{z^2}{36} = 1$

it is a Hyperboloid of Two sheets,

$\frac{x^2}{4} + y^2 - \frac{z^2}{9} = 0$ is a Cone

$\frac{x^2}{4} + y^2 - \frac{z^2}{9} = 1$ is a Hyperboloid of one sheet

Q2: Give equivalent cylindrical and spherical coordinates of the point P with rectangular coordinates $P(1,-1,\sqrt{2})$

$x = 1,\ y = -1,\ z = \sqrt{2} \Rightarrow r = \sqrt{2},\ \theta = \frac{-\pi}{4}$

cylindrical $(\sqrt{2}, \frac{-\pi}{4}, \sqrt{2})$

Spherical $(2, \frac{-\pi}{4}, \frac{\pi}{4})$.

$y = t \Rightarrow$ $x = t+2$

Q3: Determine if the limit exist $\lim\limits_{(x,y)\to(2,0)} \frac{y(x-2)}{y^2 + (x-2)^2}$

Along $x = t$, $y = t-2$ as $t \to 2$, $x \to 2$ & $y \to 0$

$\lim\limits_{t \to 0} \frac{(t-2)(t-2)}{(t-2)^2 + (t-2)^2} = \frac{1}{2}$,

Along x-axis $(y=0)$, $\lim\limits_{(x,0)\to(2,0)} \frac{0}{0 + (x-2)^2} = 0$

So the limit D.N.E.

Math 201-16 Quize #4B Sem 041

Name: Key I.D.#____________ Serial #_____

Q1: Describe the graph of the level surfaces for $f(x,y,z) = k$, where $f(x,y,z) = \frac{x^2}{9} - y^2 - \frac{z^2}{9}$ and $k = -9, 0, 1$

$k=-9$: $\frac{x^2}{9} - y^2 - \frac{z^2}{9} = -9 \Rightarrow \frac{-x^2}{81} + \frac{y^2}{9} + \frac{z^2}{81} = 1$
which is hyperboloid of one sheet.

$k=0$: $\frac{x^2}{9} - y^2 - \frac{z^2}{9} = 0$ which is a Cone.

$k=1$: $\frac{x^2}{9} - y^2 - \frac{z^2}{9} = 1$ " " hyperboloid of two sheet.

Q2: Give equivalent cylindrical and rectangular coordinates of the point P with spherical coordinates $P(2, \frac{\pi}{6}, \frac{\pi}{2})$, $\rho = 2$, $\theta = \frac{\pi}{6}$, $\phi = \frac{\pi}{2}$

$r = \rho \sin\phi = 2 \cdot 1 = 2$, $z = \rho \cos\phi = 0$

$x = r\cos\theta = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$, $y = r\sin\theta = 2 \cdot \frac{1}{2} = 1$

cylind.: $(2, \frac{\pi}{6}, 0)$

rectangular: $(\sqrt{3}, 1, 0)$

Q3: Determine if the limit exist $\lim_{(x,y,z)\to(0,0,0)} (x^2+y^2+z^2)\ln(x^2+y^2+z^2) = 0$

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Math 201-16 Quize #4C Sem 041

Name: Key I.D.#__________ Serial #_____

Q1: Describe the graph of the level surfaces for $f(x,y,z) = k$, where $f(x,y,z) = -x^2 + \frac{y^2}{9} - \frac{z^2}{4}$ and $k = -1, 0, 9$

$k=-1$: $-x^2 + \frac{y^2}{9} - \frac{z^2}{4} = -1 \Rightarrow x^2 - \frac{y^2}{9} + \frac{z^2}{4} = 1$

is Hyperboloid of one sheet.

$k=0$: $-x^2 + \frac{y^2}{9} - \frac{z^2}{4} = 0 \longrightarrow$ it is a Cone

$k=9$: $-x^2 + \frac{\cancel{x}^2}{9} - \frac{z^2}{4} = 9 \Rightarrow \frac{-x^2}{9} + \frac{y^2}{81} - \frac{z^2}{36} = 1$

is Hyperboloid of two sheets.

Q2: Give equivalent cylindrical and rectangular coordinates of the point P with spherical coordinates $P(\sqrt{3}, \frac{\pi}{4}, \frac{\pi}{4})$, $\rho = \sqrt{3}$, $\theta = \frac{\pi}{4}$, $\phi = \frac{\pi}{4}$

cylindrical: $r = \rho \sin\phi = \sqrt{3} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{3}}{\sqrt{2}}$, $x = \frac{\sqrt{3}}{\sqrt{2}} \cdot \cos\theta = \frac{\sqrt{3}}{2}$

$z = \rho\cos\phi = \sqrt{3} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{3}}{\sqrt{2}}$

$y = \frac{\sqrt{3}}{2}$

cylindrical: $\left(\frac{\sqrt{3}}{\sqrt{2}}, \frac{\pi}{4}, \frac{\sqrt{3}}{\sqrt{2}}\right)$

Rectangular: $\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{\sqrt{2}}\right)$

Q3: Determine if the limit exist $\lim_{(x,y,z)\to(0,0,0)} (x^2+y^2+z^2)\ln(x^2+y^2+z^2)$

Let $\rho^2 = x^2+y^2+z^2$, as $(x,y,z) \longrightarrow (0,0,0)$

then $\rho \longrightarrow 0^+$

$\lim_{\rho\to 0} \rho^2 \ln\rho^2 = \lim_{\rho\to 0} \frac{2\ln\rho}{1/\rho^2} \left(\frac{\infty}{\infty}\right) = \lim_{\rho\to 0} \frac{2\frac{1}{\rho}}{\frac{-2}{\rho^3}} = 0$

So ~~the~~ the limit exist and equals to 0.