

Name: _____

Key

I.D.# _____

Serial # _____

Q1: Find equation of the plane that contains the line

$$x = 2 + t, \quad y = 1 - 2t, \quad z = 2t \quad \text{and the point } P(1, 0, -1)$$

$$Q(2, 1, 0), \quad \vec{PQ} = \langle 1, 1, +1 \rangle, \quad \vec{v} = \langle 1, -2, 2 \rangle$$

$$\vec{n} = \vec{PQ} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & -2 & 2 \end{vmatrix} = \vec{i}(2+2) - \vec{j}(2-1) + \vec{k}(-2-1)$$

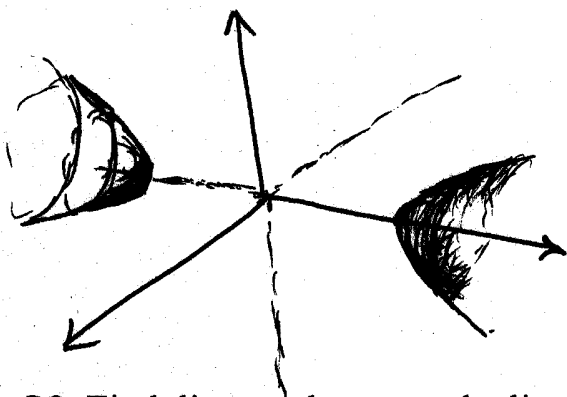
$$= 4\vec{i} - \vec{j} - 3\vec{k}$$

$$4(x-1) - (y-0) - 3(z+1) = 0$$

$$4x - y - 3z - 7 = 0$$

Q2: Identify and sketch the surface $9x^2 - 4y^2 + 9z^2 + 36 = 0$

$$-\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{4} = 1 \quad \text{Hyperboloid of two sheets}$$

Q3: Find distance between the line $L_1 : x = -1 + 2t, y = 1 - t, z = 1 - 2t$, and the plane $x + 4y - z + 2 = 0$. $P(-1, 1, 1)$

$$D = \frac{|-1 \cdot 1 + 4 \cdot 1 - 1 \cdot 1 + 2|}{\sqrt{1 + 16 + 1}} = \frac{4}{\sqrt{18}}$$

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Q1: Find equation of the plane that contains the two parallel lines

$$L_1 : x = 2 + t, y = 1 - 2t, z = 2t \text{ and } L_2 : x = 1 + 2t, y = -4t, z = 1 + 4t$$

$$\vec{v}_1 = \langle 1, -2, 2 \rangle, \quad \vec{v}_2 = \langle 2, -4, 4 \rangle$$

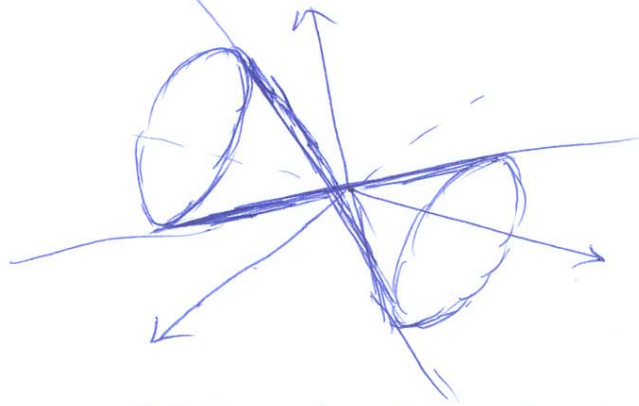
$$P_1(2, 1, 0), P_2(1, 0, 1), \quad \vec{P_1P_2} = \langle -1, -1, 1 \rangle$$

$$\vec{n} = \vec{P_1P_2} \times \vec{v}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 1 \\ 1 & -2 & 2 \end{vmatrix} = \hat{i}(-2+2) - \hat{j}(-2-1) + \hat{k}(2+1) \\ = 3\hat{j} + 3\hat{k}$$

$$\text{Eq. of the plane: } 0(x-2) + 3(y-1) + 3(z-0) = 0 \\ 3y + 3z - 3 = 0$$

Q2: Identify and sketch the surface $9x^2 - 4y^2 + 36z^2 = 0$

$$\text{it is a cone } \frac{x^2}{4} - \frac{y^2}{9} + z^2 = 0$$

Q3: Find distance between the two parallel planes $2x + 2y - z + 2 = 0$, and $-4x - 4y + 2z - 3 = 0$. $\vec{n}_1 = \langle 2, 2, -1 \rangle, \vec{n}_2 = \langle -4, -4, 2 \rangle$ let P_1 on the plane 1, $P_1(0, 0, 2)$

$$D = \frac{|0 + 0 + 4 - 3|}{\sqrt{16 + 16 + 4}} = \frac{1}{6}$$

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Q1: Find distance between the line $L_1 : x = 2 + t, y = 1 - 2t, z = 3 + t$, and the plane $3x + y - z + 1 = 0$. A point on the line $P(2, 1, 3)$

$$D = \frac{|2 - 3 + 1 \cdot 1 + 3(-1) + 1|}{\sqrt{9 + 1 + 1}} = \frac{5}{\sqrt{11}} \text{ is the distance}$$

Q2: Find equation of the plane that contains the line $x = 2 - 2t, y = t, z = 1 + 2t$ and the point $P(2, 1, 0)$

Let $Q(2, 0, 1)$
P

$$\vec{PQ} = \langle 0, -1, 1 \rangle, \quad \vec{r} = \langle -2, 1, 2 \rangle$$

$$\vec{PQ} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ -2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(0+2) + \hat{k}(0-2) \\ = -3\hat{i} - 2\hat{j} - 2\hat{k}$$

\therefore The equation of the plane is

$$-3(x-2) - 2(y-1) - 2(z-0) = 0 \quad \left\{ \begin{array}{l} -3x - 2y - 2z + 8 = 0 \end{array} \right.$$

Q3: Identify and sketch the surface $4x^2 - 4y^2 - z^2 + 16 = 0$

$$4x^2 - 4y^2 - z^2 = -16$$

$$-\frac{x^2}{4} + \frac{y^2}{4} + \frac{z^2}{16} = 1 \quad \text{Hyperboloid of one sheet}$$



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Q1: Find equation of the plane that contains the two parallel lines

$$L_1: x = -2t, y = 1+t, z = 2t \text{ and } L_2: x = 1+4t, y = -2t, z = 2-4t$$

$$P_1(0, 1, 0), P_2(1, 0, 2), \vec{v}_1 = \langle -2, 1, 2 \rangle, \vec{v}_2 = \langle 4, -2, -4 \rangle$$

$$\vec{P_1P_2} = \langle 1, -1, 2 \rangle, \vec{n} = \vec{P_1P_2} \times \vec{v}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & 1 & 2 \end{vmatrix} = \hat{i}(-4) - \hat{j}(6) + \hat{k}(-1) = -4\hat{i} - 6\hat{j} - \hat{k}$$

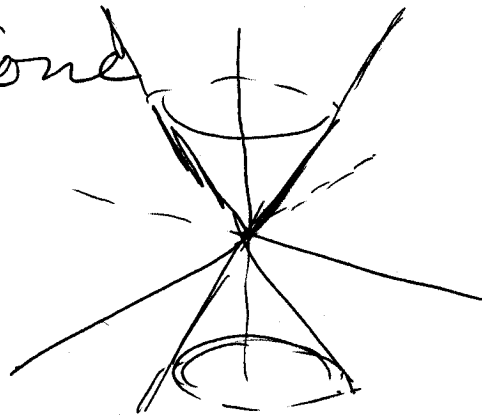
\therefore The eq. of the plane is
 $-4(x-0) - 6(y-1) - (z-0) = 0$

$$\boxed{-4x - 6y - z + 6 = 0}$$

Q2: Identify and sketch the surface $-4x^2 - 4y^2 + 16z^2 = 0$

$$\frac{x^2}{4} + \frac{y^2}{4} - z^2 = 0$$

Cone



Q3: Find distance between the two parallel planes $-x + 2y + 2z + 3 = 0$, and
 $2x - 4y - 4z + 1 = 0$. $\vec{n}_1 = \langle -1, 2, 2 \rangle, \vec{n}_2 = \langle 2, -4, -4 \rangle$

$P_1(3, 0, 0)$ on plane 1

$$D = \frac{|6 + 1|}{\sqrt{4 + 16 + 16}} = \frac{7}{6} \text{ is the distance}$$