

Name: Key

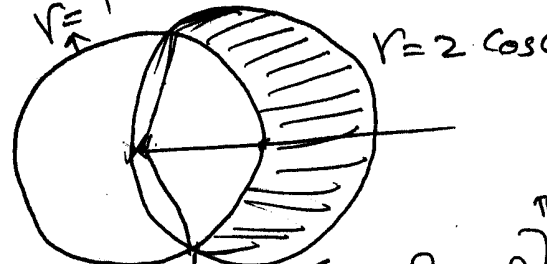
I.D.# _____

Serial # _____

Q1: Find the area inside the polar curve $r = 2 \cos \theta$ and outside the curve

$$r = 1. \quad 2 \cos \theta = 1 \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$A = \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} [(2 \cos \theta)^2 - 1^2] d\theta$$



$$= \int_0^{\pi/3} [4 \cos^2 \theta - 1] d\theta = \int_0^{\pi/3} (2 \cos 2\theta + 2 - 1) d\theta = \sin 2\theta + \theta \Big|_0^{\pi/3}$$

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{3}$$

Q2: Find equation of the sphere that is centered at $(-3, 2, 1)$, and tangent to the y -axis.

tangent to y -axis $\Rightarrow r = \sqrt{9+1} = \sqrt{10}$

$$\therefore (x+3)^2 + (y-2)^2 + (z-1)^2 = 10$$

Q3: Find the magnitude of $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$, where $\mathbf{u} = \langle -2, 0, 3 \rangle$ and $\mathbf{v} = \langle 2, -1, 1 \rangle$.

$$\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle 0, -1, 4 \rangle, \quad \vec{\mathbf{u}} - \vec{\mathbf{v}} = \langle -4, 1, 2 \rangle$$

$$\|\vec{\mathbf{u}} + \vec{\mathbf{v}}\| = \sqrt{1+16} = \sqrt{17},$$

$$\|\vec{\mathbf{u}} - \vec{\mathbf{v}}\| = \sqrt{16+1+4} = \sqrt{21}$$