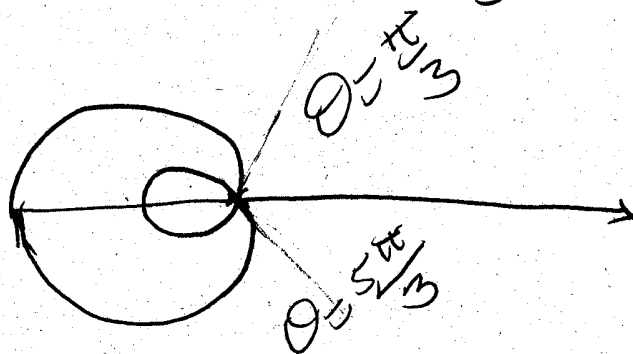
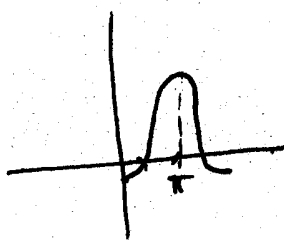


Name: Key I.D.# _____ Serial # _____

Q1: Sketch the graph of the polar curve $r = 1 - 2\cos\theta$, show the angles where the graph passes through the pole.

$$r=0 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$



Q2: Find the slope of the tangent line to the parametric curve $x = \sin t$,

$$y = \cos t \text{ at } t = \frac{\pi}{6}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\sin t}{\cos t} \Big|_{t=\frac{\pi}{6}} = \frac{-1/2}{\sqrt{3}/2} = \frac{-1}{\sqrt{3}}$$

Q3: Find the angles where the graph $r = \sin\theta$, $0 \leq \theta \leq \pi$ has vertical tangent.

$$\frac{dx}{d\theta} = 0 \text{ and } \frac{dy}{d\theta} \neq 0$$

$$x = r \cos\theta = \sin\theta \cos\theta, \quad y = r \sin\theta = \sin^2\theta$$

$$\frac{dx}{d\theta} = \cos^2\theta - \sin^2\theta, \quad \frac{dy}{d\theta} = 2\sin\theta \cos\theta$$

$$\frac{dx}{d\theta} = 0 \Rightarrow \cos^2\theta - \sin^2\theta = 0 \Rightarrow 1 - 2\sin^2\theta = 0 \Rightarrow \sin\theta = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4} \checkmark$$

$$\frac{dy}{d\theta} \Big|_{\theta=\frac{\pi}{4}} = 2 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = 1, \quad \frac{dy}{d\theta} \Big|_{\theta=\frac{3\pi}{4}} \neq 0$$

