King Fahd University of Petroleum and Minerals

Department of Mathematics

Math 102 Sem II (052)

Final Exam B | Sun 4 / 6 / 2006 | Time $2\frac{1}{2}$ hours

Name: ________________________ I.D.#: ________________ Serial #: ______

Section # [17] [21]

Answer all the questions

For the solving part show all of your work

For the multiple choice part mark your answer in the following table

Question #	a	b	c	d	e
1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e

Question #	15	16	17	18	19	20	21	Total
GRADE	/4	/4	/4	/4	/4	/4	/4	/ 28

1. If the region bounded by the curves $y = x^2$, $x - axis$, and $x = 1$ is revolving about $x = -2$, then the volume is expressed by the integral

a. $V = 2\pi \int_0^2 (x-1)x^2\,dx$

b. $V = 2\pi \int_0^1 (x^2-2)x\,dx$

c. $V = 2\pi \int_0^2 (x+1)x^2\,dx$

d. $V = 2\pi \int_0^1 (x+2)x^2\,dx$

e. $V = 2\pi \int_0^2 (x^2-1)x\,dx$

2. The arclength of the curve $y = \frac{2}{3}x^{\frac{3}{2}}$ for $x = 0$ to $x = 3$ is equal to :

a. $\frac{14}{3}$

b. $\frac{26}{7}$

c. $\frac{124}{15}$

d. $\frac{15}{4}$

e. $\frac{22}{5}$

3. One of the following is TRUE

a. $\int \frac{2}{x^2-1}\,dx = \int \frac{dx}{x-1} - \int \frac{dx}{x+1}$

b. The series $\sum_{n=0}^{\infty}\left(\frac{3}{2}\right)^n = \frac{1}{1-\frac{3}{2}}$

c. $\int_0^1 \sqrt{1-x^2}\;dx = \frac{\pi}{2}$

d. The integral $\int_1^{\infty} \frac{1}{x}\,dx$ is convergent.

e. The sequence $\left\{\frac{\sin n}{n}\right\}_{n=1}^{\infty}$ is divergent

4. The area of the region bounded by $y = \ln x$, $x - axis$, and $x = e$ is :

Notation **a**. e

b. $\frac{1}{2}$

c. $\ln(e-1)$

d. $e-1$

e. 1

5. $\int_{\frac{6}{\pi}}^{\frac{3}{\pi}} \frac{1}{x^2 \sin \frac{1}{x}}\, dx$

a. $2 - \frac{2}{\sqrt{3}}$

b. $\frac{1}{2} - \frac{2}{\sqrt{3}}$

c. $\frac{\sqrt{3}}{2} - \ln 2$

d. $\ln\sqrt{3} + 2$

e. $\ln\sqrt{3} + \ln(2 - \sqrt{3})$

6. $\int_0^2 \frac{dx}{(x-1)^2} =$

a. 2

b. $\frac{1}{2}$

c. Divergent

d. -1

e. -2

7. $\int_0^{\frac{\pi}{3}} \tan^3 x \sec^4 x \, dx =$

a. $\frac{9}{2}$

b. $\frac{9\sqrt{3}}{2}$

c. $\frac{27}{2}$

d. $\frac{27}{4}$

e. $\frac{9}{4}$

8. $\frac{d}{dx} \int_{\sqrt{x}}^{\frac{1}{x}} \frac{\tan t}{t^2 + 1} \, dt$

a. $\ln \frac{1}{x} - \ln \sqrt{x}$

b. $\frac{-\tan \frac{1}{x}}{1 + x^2} - \frac{\tan \sqrt{x}}{2x\sqrt{x} + 2\sqrt{x}}$

c. $\frac{-1}{(1 + x^2)^2}$

d. $\tan^{-1} \frac{1}{x} - \tan^{-1} \sqrt{x}$

e. $\frac{\tan^2 \frac{1}{x}}{2} - \frac{\tan^2 \sqrt{x}}{2}$

9. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n n^2}{10^n}$ is equal to:

a. 5

b. 10

c. $\sqrt{10}$

d. 1

e. 2

10. $\int_0^1 \sin^{-1} x \, dx =$

a. $\frac{\pi}{2}$

b. $\frac{\pi}{2} - 1$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{4} + 1$

e. 1

11. $\int_{-\frac{\pi}{2}}^{0} \frac{\cos\theta}{\sin^2\theta + 2\sin\theta + 2}\,d\theta$

a. $\ln 5 - 1$

b. $\ln 5 - \ln 2$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{2}$

e. 1

12. $\int_{0}^{\ln 2} \frac{dx}{\sqrt{e^{2x} - 1}} =$

a. $\frac{\sqrt{e^4 - 1}}{2}$

b. $\frac{\pi}{4}$

c. 1

d. $2\sqrt{e^4 - 1}$

e. $\frac{\pi}{3}$

13. Let the series $\sum_{k=1}^{\infty}\left[\frac{1}{2k-1}-\frac{1}{2k+1}\right]$ then the n th partial sum of the series S_n is equal to:

a. $S_n = \frac{2}{3} - \frac{1}{2n+1}$

b. $S_n = \frac{3}{4} - \frac{1}{2n+1}$

c. $S_n = 1 - \frac{1}{2n+1}$

d. $S_n = \frac{3}{2} - \frac{1}{2n+1}$

e. $S_n = \frac{1}{2} - \frac{1}{2n+1}$

14. If the parametric curve $x = \cos^2 t$, $y = \sin^2 t$, $0 \le t \le \frac{\pi}{2}$, is revolving about the $x-axis$, then the surface area is equal to :

a. $\frac{\pi}{2}$

b. $\frac{\pi}{\sqrt{2}}$

c. π

d. $\pi\sqrt{2}$

e. 2π

Solving Part (Show all of your work)

15. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$

16. $\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\, dx =$

17. $\displaystyle\int_0^{\ln 2} \frac{\tanh x}{1+\sinh^2 x}\,dx =$

18. Find the volume of revolution if the region between the curve $y = \cos x$, and $x-axis$ from $x = 0$ to $x = \frac{\pi}{2}$ is revolving about the $x-axis$

19. Determine if the series $\sum_{n=0}^{\infty} \frac{3^n + 2^n}{7(5)^n}$ is convergent or divergent , give the sum if convergent.

20. Use the fact that $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$, for $|x| < 1$, find the power series representation of $\tan^{-1}(2x)$, $|x| < \frac{1}{2}$

21. Detrmine if the sequence $\left\{\frac{n!}{n^n}\right\}_{n=1}^{\infty}$ is convergent or divergent