

King Fahd University of Petroleum and Minerals  
 Department of Mathematics  
 Math 102 Sem II (052)

Final Exam **B**

Sun 4 / 6 / 2006

Time  $2\frac{1}{2}$  hours

Name: \_\_\_\_\_ I.D.#: \_\_\_\_\_ Serial #: \_\_\_\_\_

Section # 17 21

Answer all the questions

For the solving part show all of your work

For the multiple choice part mark your answer in the following table

Question #	a	b	c	d	e
1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e
13	a	b	c	d	e
14	a	b	c	d	e

Question #	15	16	17	18	19	20	21	Total
GRADE	/4	/4	/4	/4	/4	/4	/4	/ 28

1. If the region bounded by the curves  $y = x^2$ ,  $x$  axis, and  $x = 1$  is revolving about  $x = 2$ , then the volume is expressed by the integral

- a.  $V = 2 \int_0^2 x + 1 \, dx$
- b.  $V = 2 \int_0^1 x^2 + 2x \, dx$
- c.  $V = 2 \int_0^2 x + 1 \, dx$
- d.  $V = 2 \int_0^1 x + 2x^2 \, dx$
- e.  $V = 2 \int_0^2 x^2 + 1 \, dx$
2. The arclength of the curve  $y = \frac{2}{3}x^{\frac{3}{2}}$  for  $x = 0$  to  $x = 3$  is equal to :

- a.  $\frac{14}{3}$
- b.  $\frac{26}{7}$
- c.  $\frac{124}{15}$
- d.  $\frac{15}{4}$
- e.  $\frac{22}{5}$
3. One of the following is TRUE

- a.  $\int \frac{2}{x^2 + 1} dx = \frac{dx}{x + 1} + \frac{dx}{x - 1}$
- b. The series  $\sum_{n=0}^{\infty} \frac{3}{2} \left(\frac{1}{3}\right)^n$  is convergent.
- c.  $\int_0^1 \sqrt{1 - x^2} \, dx = \frac{\pi}{2}$
- d. The integral  $\int_1^{\infty} \frac{1}{x} dx$  is convergent.
- e. The sequence  $\left\{ \frac{\sin n}{n} \right\}_{n=1}^{\infty}$  is divergent
4. The area of the region bounded by  $y = \ln x$ ,  $x$  axis, and  $x = e$  is :

- Notation**
- a.  $e$
- b.  $\frac{1}{2}$
- c.  $\ln e - 1$
- d.  $e - 1$
- e. 1
5.  $\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{1}{x^2 \sin \frac{1}{x}} dx$
- a.  $2 - \frac{2}{\sqrt{3}}$
- b.  $\frac{1}{2} - \frac{2}{\sqrt{3}}$
- c.  $\frac{\sqrt{3}}{2} - \ln 2$
- d.  $\ln \sqrt{3} - 2$
- e.  $\ln \sqrt{3} - \ln 2 - \sqrt{3}$
6.  $\int_0^2 \frac{dx}{x + 1} - 2$

- a. 2  
 b.  $\frac{1}{2}$   
 c. Divergent  
 d. 1  
 e. 2
7.  $\int_0^{\sqrt{3}} \tan^3 x \sec^4 x \, dx$
- a.  $\frac{9}{2}$   
 b.  $\frac{9\sqrt{3}}{2}$   
 c.  $\frac{27}{2}$   
 d.  $\frac{27}{4}$   
 e.  $\frac{9}{4}$
8.  $\frac{d}{dx} \int_{\sqrt{x}}^{\frac{1}{x}} \frac{\tan t}{t^2 - 1} dt$
- a.  $\ln \frac{1}{x} - \ln \sqrt{x}$   
 b.  $\frac{\tan \frac{1}{x}}{1 - x^2} - \frac{\tan \sqrt{x}}{2x\sqrt{x} - 2\sqrt{x}}$   
 c.  $\frac{1}{1 - x^2 - 2}$   
 d.  $\tan^{-1} \frac{1}{x} - \tan^{-1} \sqrt{x}$   
 e.  $\frac{\tan^2 \frac{1}{x}}{2} - \frac{\tan^2 \sqrt{x}}{2}$
9. The radius of convergence of the power series  $\sum_{n=1}^{\infty} \frac{x^{n-1}}{10^n}$  is equal to:
- a. 5  
 b. 10  
 c.  $\sqrt{10}$   
 d. 1  
 e. 2
10.  $\int_0^1 \sin^{-1} x \, dx$
- a.  $\frac{1}{2}$   
 b.  $\frac{1}{2} - 1$   
 c.  $\frac{1}{4}$   
 d.  $\frac{1}{4} - 1$   
 e. 1
11.  $\int_{\frac{1}{2}}^0 \frac{\cos^{-1} \sin \frac{d}{2}}{\sin^2 \frac{d}{2}} d$
- a.  $\ln 5 - 1$   
 b.  $\ln 5 - \ln 2$   
 c.  $\frac{1}{4}$

- d.  $\frac{1}{2}$   
 e.  $\frac{1}{\ln 2}$
12.  $\int_0^1 \frac{dx}{\sqrt{e^{2x} - 1}}$
- a.  $\frac{\sqrt{e^4 - 1}}{2}$   
 b.  $\frac{1}{4}$   
 c.  $\frac{1}{2}$   
 d.  $2\sqrt{e^4 - 1}$   
 e.  $\frac{1}{3}$
13. Let the series  $\sum_{k=1}^{\infty} \left[ \frac{1}{2k-1} - \frac{1}{2k+1} \right]$  then the  $n$ th partial sum of the series  $S_n$  is equal to:
- a.  $S_n = \frac{2}{3} - \frac{1}{2n+1}$   
 b.  $S_n = \frac{3}{4} - \frac{1}{2n+1}$   
 c.  $S_n = 1 - \frac{1}{2n+1}$   
 d.  $S_n = \frac{3}{2} - \frac{1}{2n+1}$   
 e.  $S_n = \frac{1}{2} - \frac{1}{2n+1}$
14. If the parametric curve  $x = \cos^2 t$ ,  $y = \sin^2 t$ ,  $0 \leq t \leq \frac{\pi}{2}$ , is revolving about the  $x$  axis, then the surface area is equal to:
- a.  $\frac{1}{2}$   
 b.  $\frac{1}{\sqrt{2}}$   
 c.  $\frac{1}{2}$   
 d.  $\sqrt{2}$   
 e. 2
- Solving Part (Show all of your work)
15. Find the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$
16.  $\int \frac{1}{\sqrt[3]{x} \sqrt{x}} dx$
17.  $\int_0^{\ln 2} \frac{\tanh x}{1 + \sinh^2 x} dx$
18. Find the volume of revolution if the region between the curve  $y = \cos x$ , and  $x$  axis from  $x = 0$  to  $x = \frac{\pi}{2}$  is revolving about the  $x$  axis
19. Determine if the series  $\sum_{n=0}^{\infty} \frac{3^n - 2^n}{7 \cdot 5^n}$  is convergent or divergent, give the sum if convergent.
20. Use the fact that  $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$ , for  $|x| < 1$ , find the power series representation of  $\tan^{-1} 2x$ ,  $|x| < \frac{1}{2}$
21. Determine if the sequence  $\left\{ \frac{n!}{n^n} \right\}_{n=1}^{\infty}$  is convergent or divergent

Math 102 Sem II (052) Second Major Exam Wed 26 / 4 / 2006 Time 1 1/4 hours

Answer all the questions  
Show all of your work

Question #	1	2	3	4	5	6	7	8	Total
Grade	/5	/5	/5	/5	/5	/5	/5	/5	/40

- Find the arclength of the parametric curve  $y = e^t \sin t, x = e^t \cos t$  from  $t = 0$  to  $t = 1$
- $\int x \tan^{-1} x \, dx$       8.  $\int \frac{x^2}{\sqrt{4x-x^2}} \, dx$
- Find the surface area generated if the curve  $y = \cosh x, 0 \leq x \leq 2$  is revolving about the  $y$  axis.
- $\int \frac{2}{x(x-1)^2} \, dx$       6.  $\int_0^1 \frac{e^x}{\sqrt{e^{2x}-1}} \, dx$       7.  $\int \tan^{\frac{3}{2}} x \sec^4 x \, dx$
- Determine if the sequence  $\frac{3 \cdot 2^n}{3^n} \cdot n \cdot n^{-1}$  is convergent or divergent, if convergent find the limit.

Math 102 Sem II (052) First Major Exam Wed 22 / 3 / 2006 Time 1 1/4 hours

Question #	1a	1b	1c	2	3	4	5	6	7	Total
Grade	/4	/4	/4	/4	/5	/5	/5	/4	/5	/40

- Integrate each of the following:
  - $\int_1^4 \frac{x^3 - 3x - 2}{\sqrt{x}} \, dx$
  - $\int \frac{\cos 2x}{\sqrt{4 - 3 \sin 2x}} \, dx$
  - $\int x^{-1} \cdot 23x^2 \, dx$
- Find the derivative  $\frac{d}{dx} \int_{\tan x}^{x^2} \frac{t-3}{t^2-1} \, dt$
- Find the area of the region bounded by  $y = x^2, y = 4x - 4$ , and  $y = 0$
- Find the volume generated if the region bounded by the curves  $y = x^3, x = 1, y = 0$  is revolving about the  $x$  axis
- Express the Riemann Sum  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n}{n^2 - k^2}$  over the interval  $[0, 1]$  as a definite integral and solve it
- Find the volume generated if the region bounded by the curves  $y = x^2, y = x - 2$ , is revolving about the line  $x = 1$
- $\int_0^{\ln 2} \frac{1}{1 - e^{-x}} \, dx$