

Name: _____ I.D.#: _____ Serial
#: _____

Section #

14

20

Answer all the questions

For the solving part show all of your work

For the multiple choice part indicate your choice in the given table (one choice only)

Question #	a	b	c	d	e
1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e

Question #	13	14	15	16	17	18	19	20	
Grade	/4	/4	/5	/4	/4	/4	/4	/5	

Total	/ 70
-------	------

1. Part I The Multiple Choice

2. $\lim_{w \rightarrow 1} \frac{2^w - 2}{w - 1}$
- 2
 - $\ln 2$
 - $2 \ln 2$
 - 1
 - 0
3. Let $f(x) = x^4 - 2x^2 + 7$, then one of the following is FALSE:
- The function is increasing over the interval $(0, 1)$.
 - The function is increasing over the interval $(-1, 1)$.
 - The function is decreasing over the interval $(0, 1)$.
 - The function has relative maximum at $x = 0$.
 - The function has relative minimum at $x = 1$.
4. If $y = \tan^{-1} \frac{x}{2}$, then $\frac{dy}{dx}$ is equal to:
- $\frac{1}{2} \csc \frac{x}{2} \cot \frac{x}{2}$
 - $\frac{1}{\sqrt{4 - x^2}}$
 - $\frac{1}{2} \sec^2 \frac{x}{2}$
 - $\frac{1}{\sqrt{x^2 - 4}}$
 - $\frac{2}{x^2 - 4}$
5. The equation of the tangent line to the graph of $f(x) = 2x^3 - 1$ at $x = 1$ is:
- $y = 6x - 2$
 - $y = 6x - 3$
 - $y = 6x - 4$

- d. $y = 6x - 5$
- e. $y = 6x + 1$
6. $\lim_{x \rightarrow 3} \sqrt{x^2 - 3x} = x$
- a. $\frac{3}{2}$
- b. $\frac{3}{4}$
- c.
- d.
- e. 0
7. $\frac{d}{dx} \log_x 5$
- a. $\frac{\ln 5}{\ln x}$
- b. $\frac{\ln 5}{x \ln x^2}$
- c. $5 \ln x$
- d. $\frac{\ln x}{\ln 5}$
- e. $\frac{x \ln x}{\ln 5}$
8. One of the following is TRUE :
- a. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \frac{1}{2}$
- b. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$
- c. $\lim_{x \rightarrow 5} \frac{2x - 3}{5 - x} = \frac{2}{5}$
- d. $\lim_{x \rightarrow 2} \sqrt{x^2 - 5} = 1$
- e. $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 1$
9. The equation of the tangent line to the graph of $y = \sqrt{x - 1}$ that passes through the origin is :

a. $y = 2x$

b. $y = x$

c. $y = \frac{x}{2}$

d. $y = \frac{3}{2}x$

e. $y = \frac{2}{3}x$

10. Let $f(x) = 3x^{\frac{2}{3}} - x + 5$, $x \in [1, 8]$, then the absolute maximum of $f(x)$ is

a. 0

b. $9\sqrt{2}$

c. $9\sqrt[3]{4}$

d. 18

e. 36

11. Let $f(x) = 5x - \sin 2x$, $\frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$. The function $f(x)$ has an inverse f^{-1} , then $\frac{d}{dx}f^{-1}(0)$ is equal to :

a. 0

b. $\frac{1}{5}$

c. 5

d. $\frac{1}{3}$

e. 3

12. Let $f(x) = \begin{cases} \frac{1}{x} - \frac{1}{2} & \text{if } x < 2 \\ k & \text{if } x = 2 \end{cases}$. In order for f to be continuous at $x = 2$, the value of k should be :

a. $\frac{1}{2}$

b. $-\frac{1}{2}$

c. $-\frac{1}{4}$

d. $\frac{1}{4}$

e. 1

13. One of the following is FALSE :

a. $\frac{d}{dx} \sin 2x = 2 \cos 2x$

b. $\frac{d}{dx} \tan x = \sec^2 x$

c. $\frac{d}{dx} \sqrt{x^2 - 1} = \frac{x}{\sqrt{x^2 - 1}}$

d. $\frac{d}{dx} 3^x = 3^x \ln 3$

e. $\frac{d}{dx} \ln \sqrt{x} = \frac{1}{2\sqrt{x}}$

Part II Solving **show all of your work**

14. Let $f(x) = x^3 \sec(x^2 - 1)^5$, then find $\frac{df}{dx}$

15. Find the dimensions of the right circular cylinder of largest volume that can be inscribed in a sphere of radius 15.

16. $\lim_{h \rightarrow 0} \frac{1 - \cos 3h}{\cos 5h - 1}$

17. Use local linear approximation to show that $\frac{1}{1 - 2x^5} \approx 1 + 10x$

18. A spherical balloon is to be deflated so that its volume V is decreasing at a rate of $4 \text{ ft}^3/\text{min}$. How fast is the diameter S of the balloon decreasing when the radius r is 3 ft ?
Where $V = \frac{4}{3} r^3 = \frac{\pi}{6} S^3$

19. Let $\cos y^2 = y^3 x - 2x$, where y is a differentiable function of x defined implicitly, find $\frac{dy}{dx}$

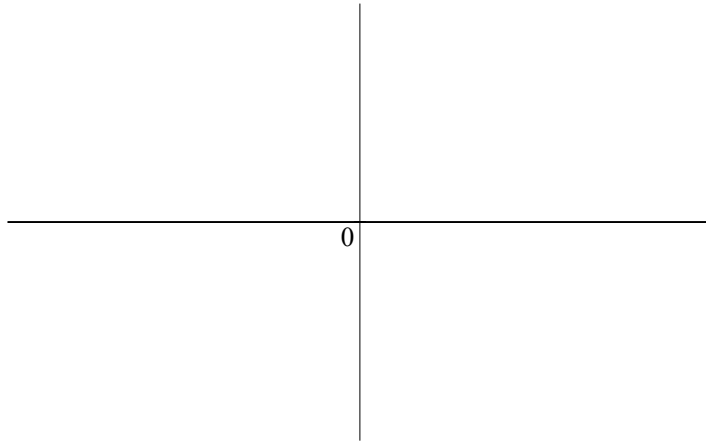
20. Use The Mean Value Theorem to show that $|\sin t - \sin v| \leq |t - v|$, where $0 < v < t$.

21. Let $f(x) = xe^{-x^2}$, $f'(x) = e^{-x^2} - 2x^2$, and $f''(x) = 2xe^{-x^2} - 3 - 2x^2$

Sketch the graph of the function $f(x)$ discussing the following:

- Symmetries:
- x and y-intercepts:
- Intervals where $f(x)$ increase and decrease:
- relative extrema:

- Concavity:
- Inflection points:
- Asymptotes



King Fahd University of Petroleum and Minerals
 Department of Mathematics
 Math 101 Sem I 2005-2006

First Major Exam

Sunday 9 / 10 / 2005

Time $1\frac{1}{4}$ hours

Name: _____ I.D.#: _____ Serial #: _____

Section # 14 20

Answer all the questions

Show all of your work

Question #	1	2	3	4	5	6	7	8	Total /40
Grade	/5	/5	/5	/5	/5	/5	/5	/5	

1. Use basic definition to find $f'(2)$, if $f(x) = \frac{1}{2} - 2x^2$
2. Use basic definition to find $f'(x)$, if $f(x) = \frac{1}{x-1}$

3. Given $\epsilon > 0$, find a suitable $\delta > 0$, to show $\lim_{x \rightarrow 2} (9 - 2x) = 5$

4. $\lim_{x \rightarrow 0} \frac{e^x}{e^{-x} e^x}$

5. Let $f(x) = \begin{cases} \frac{\sin(x-2)}{x^2-4} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$

6. Let $f(x) = \begin{cases} x \cos \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$, show that $f(x)$ is continuous, but it is not differentiable at $x = 0$

7. To show that $\lim_{x \rightarrow 1} \frac{4x}{2x-1} = 2$, let $\epsilon = 0.00001$, then find a suitable $M > 0$, so that if $|x - 1| < M$, then $|\frac{4x}{2x-1} - 2| < \epsilon$

8. For each of the following give a short answer in the assigned space:

a. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

b. $\lim_{x \rightarrow 1} (3x - 2)$

c. $\lim_{x \rightarrow 0} \ln(3x)$

d. $\lim_{x \rightarrow 1} \frac{2x^2 - 3x + 7}{7x^2 - x + 1}$

e. The domain of continuity of $f(x) = \frac{1}{1-x}$ is equal to

Math 101 Sem I 2005-2006
 Second Major Exam Tuesday 6 / 12 / 2005 Time 1 $\frac{1}{4}$ hours

Name: _____ I.D.#: _____ Serial #: _____

Section # 14 20

Answer all the questions
 Show all of your work

Question #	1	2	3	4	5	6	7	8	Total /40
Grade	/5	/5	/5	/5	/5	/5	/5	/5	

1. Use implicit differentiation to find $\frac{dy}{dx}$ if $3xy = \tan y - 2$
2. $\lim_{x \rightarrow 0} (1 - \tan x)^{\csc x}$
3. Find $f'(x)$, if $f(x) = \ln 10^x \cos x$
4. $\lim_{x \rightarrow 0} \frac{\frac{1}{x}}{\frac{1}{e^x - 1}}$
5. Let $f(x) = x^2 e^x$, $x > 0$. f is 1-1 and let $f^{-1}(x) = y$, then find $\frac{dy}{dx}$, and $\frac{d}{dx} \frac{1}{e}$
6. Let $f(x) = e^x \sec^2 x^{\frac{3}{5}}$, find $\frac{df}{dx}$ (do not simplify)
7. Use local linear approximation to estimate $\sin 31^\circ$, let $f(x) = \sin x$ and $x = \frac{\pi}{6}$
8. An aircraft is climbing at a 30° angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/h?