

Name: _____ I.D.#: _____ Serial
#: _____

Section #

14

20

Answer all the questions

For the solving part show all of your work

For the multiple choice part indicate your choice in the given table (one choice only)

Question #	a	b	c	d	e
1	a	b	c	d	e
2	a	b	c	d	e
3	a	b	c	d	e
4	a	b	c	d	e
5	a	b	c	d	e
6	a	b	c	d	e
7	a	b	c	d	e
8	a	b	c	d	e
9	a	b	c	d	e
10	a	b	c	d	e
11	a	b	c	d	e
12	a	b	c	d	e

Question #	13	14	15	16	17	18	19	20	
Grade	/4	/4	/5	/4	/4	/4	/4	/5	

Total	/ 70
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1. Part I The Multiple Choice

2. $\lim_{w \rightarrow 1} \frac{2^w - 2}{w - 1}$

- a. 2
- b. $\ln 2$
- c. $2 \ln 2$
- d. 1
- e. 0

3. Let $f(x) = x^4 - 2x^2 - 7$, then one of the following is FALSE:

- a. The function is increasing over the interval $[-1, 1]$
- b. The function is increasing over the interval $(-1, 1)$
- c. The function is decreasing over the interval $(0, 1)$
- d. The function has relative maximum at $x = 0$
- e. The function has relative minimum at $x = 1$

4. If $y = \tan^{-1} \frac{x}{2}$, then $\frac{dy}{dx}$ is equal to :

a. $\frac{1}{2} \csc \frac{x}{2} \cot \frac{x}{2}$

b. $\frac{1}{\sqrt{4-x^2}}$

c. $\frac{1}{2} \sec^2 \frac{x}{2}$

d. $\frac{1}{\sqrt{x^2-4}}$

e. $\frac{2}{x^2-4}$

5. The equation of the tangent line to the graph of $f(x) = 2x^3 - 1$ at $x = 1$ is :

a. $y = 6x - 2$

b. $y = 6x - 3$

c. $y = 6x - 4$

d. $y = 6x - 5$

e. $y = 6x - 1$

6. $\lim_x \sqrt{x^2 - 3x} - x$

a. $\frac{3}{2}$

b. $\frac{3}{4}$

c.

d.

e. 0

7. $\frac{d}{dx} \log_x 5$

a. $\frac{\ln 5}{\ln x}$

b. $\frac{\ln 5}{x \ln x^2}$

c. $5 \ln x$

d. $\frac{\ln x}{\ln 5}$

e. $\frac{x \ln x}{\ln 5}$

8. One of the following is TRUE :

a. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \frac{1}{2}$

b. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$

c. $\lim_{x \rightarrow 5} \frac{2x - 3}{5 - x} = \frac{2}{5}$

d. $\lim_{x \rightarrow 2} \sqrt{x^2 - 5} = 1$

e. $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 1$

9. The equation of the tangent line to the graph of $y = \sqrt{x - 1}$ that passes through the origin is :

a. $y = 2x$

b. $y = x$

c. $y = \frac{x}{2}$

d. $y = \frac{3}{2}x$

e. $y = \frac{2}{3}x$

10. Let $f(x) = 3x^{\frac{2}{3}} - x + 5$, $x \in [1, 8]$, then the absolute maximum of $f(x)$ is
- a. 0
 - b. $9\sqrt{2}$
 - c. $9\sqrt[3]{4}$
 - d. 18
 - e. 36
11. Let $f(x) = 5x - \sin 2x$, $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$. The function $f(x)$ has an inverse f^{-1} , then $\frac{d}{dx} f^{-1}(0)$ is equal to :
- a. 0
 - b. $\frac{1}{5}$
 - c. 5
 - d. $\frac{1}{3}$
 - e. 3

12. Let $f(x) = \begin{cases} \frac{1}{x} - \frac{1}{2} & \text{if } x > 2 \\ k & \text{if } x = 2 \end{cases}$. In order for f to be continuous at $x = 2$, the value of k should be :

a. $\frac{1}{2}$

b. $\frac{1}{2}$

c. $\frac{1}{4}$

d. $\frac{1}{4}$

e. 1

13. One of the following is FALSE :

a. $\frac{d}{dx} \sin 2x = 2 \cos 2x$

b. $\frac{d}{dx} \tan x = \sec^2 x$

c. $\frac{d}{dx} \sqrt{x^2 - 1} = \frac{x}{\sqrt{x^2 - 1}}$

d. $\frac{d}{dx} 3^x = 3^x \ln 3$

e. $\frac{d}{dx} \ln \sqrt{x} = \frac{1}{2\sqrt{x}}$

Part II Solving show all of your work

14. Let $f(x) = x^3 \sec(x^2 - 1)^5$, then find $\frac{df}{dx}$
15. Find the dimensions of the right circular cylinder of largest volume that can be inscribed in a sphere of radius 15.

16. $\lim_{h \rightarrow 0} \frac{1 - \cos 3h}{\cos 5h - 1}$

17. Use local linear approximation to show that $\frac{1}{1 - 2x^5} \approx 1 + 10x$

18. A spherical balloon is to be deflated so that its volume V is decreasing at a rate of $4 \text{ ft}^3/\text{min}$. How fast is the diameter S of the balloon decreasing when the radius r is 3 ft ?
Where $V = \frac{4}{3} r^3 = \frac{\pi}{6} S^3$

19. let $\cos y^2 = y^3 x - 2x$, where y is a differentiable function of x defined implicitly, find $\frac{dy}{dx}$

20. Use The Mean Value Theorem to show that $|\sin t - \sin v| \leq |t - v|$, where $0 \leq v < t$.

21. Let $f(x) = xe^{x^2}$, $f(x) = e^{x^2} - 1 - 2x^2$, and $f(x) = 2xe^{x^2} - 3 - 2x^2$

Sketch the graph of the function $f(x)$ discussing the following:

- Symmetries:
- x and y-intercepts:
- Intervals where $f(x)$ increase and decrease:
- relative extrema:
- Concavity:
- Inflection points:
- Asymptotes

