

**Final Exam – Math 101**

**Section 22**

**Q1.** Let  $g(x) = x^2 \sqrt{8 - x^3}$ . Find  $g'(x)$  and  $g'(1)$ .

*6 points*

**Q2.** Determine whether the limit exists or not:

$$\lim_{x \rightarrow 2} \frac{x^3 - 2^3}{x - 2}.$$

*5 points*

**Q3.**  $\int \frac{\cos 2x}{\sqrt{1 - \sin 2x}} dx$

*5 points*

- Q4.** A stone is thrown upward ,it takes 8 seconds to reach the surface of the ground again
- Find the maximum height the stone could attain.
  - What is the speed of the stone 2 seconds after it is thrown?
- use  $g = 32 \text{ ft/s}^2$

*6 points*

- Q5.** Find the values of  $a$  and  $b$  so that the following function is continuous at  $x = 2$

$$f(x) = \begin{cases} x^2 - ax - 2 & \text{if } x < 2 \\ 5 - b & \text{if } x = 2 \\ bx - 4 & \text{if } x > 2 \end{cases}$$

*6 points*

**Q6.** Sketch the function with the following properties:

$$f(x) = \frac{x^2}{x^2 - x - 6} - \frac{x^2}{x^2 - 3x - 2}, \quad f(x) < 0, \quad \text{if } x < 0$$

$$f(x) = \frac{x^2 - 12x}{x^2 - x - 6^2}, \text{ and } f(x) > 0, \quad \text{if } x > 0 \text{ or } x > 12$$

$$f(x) = \frac{2x^3 - 36x^2 - 72}{x^2 - x - 6^3}, \quad \text{and } f(x) < 0, \quad \text{if } x > 18.11$$

$$f(0) = 0, \quad f(12) = 0.960, \quad f(18.11) = 0.964$$

Show asymptotes and give table of signs of  $f(x)$  &  $f'(x)$ .

6 points

**Q7.** Evaluate the integrals :

*5 points each*

a.  $\int_1^2 \left( \frac{1}{x} - \frac{1}{x^2} \right) dx$

b.  $\int_3^4 \sqrt{9 - x^2} dx$

**Q8.** .

*6 points*

**Q9.** Let  $f(x) = x\sqrt{4-x^2}$ . Find the local extrema of  $f(x)$  and the intervals where  $f(x)$  is increasing or decreasing, and sketch the graph of  $f(x)$ .

*7 points*

**Q10.** If  $f(x) = 2\sin x - \cos 2x$ , find equations of tangent and normal lines to the graph of  $f(x)$  at the point  $(\frac{\pi}{4}, \frac{2}{\sqrt{2}})$ .

*5 points*

**Q11.**  $\lim_{x \rightarrow 0} \frac{3x - 1 - \cos^2 x}{\sin x}$

*6 points*

**Q12.** A merchant sells shoes at 100 Riyals per pair if fewer than 10 pairs ordered .If 10 or more pairs are ordered ( up to 50 ) , the price per pair is reduced by one Riyal times the number ordered . What size order will produce the maximum amount of money for the wholesaler ?

*6 points*

**Q13.**

*6 points*