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Math 132

PROBLEMS ON LIMIT AND CONTINUITY

Problem 1: If the limit exists find it. If it does not exist show why and use the symbols ∞ or $-\infty$ whenever appropriate.

(1) $\lim_{x\to 2} \dfrac{x^2-5x+6}{x^2-7x+10}$

(2) $\lim_{x\to -3} \dfrac{x+3}{x^2-9}$

(3) $\lim_{x\to -3^+} \sqrt{9-x^2}$, $\lim_{x\to -3^-} \sqrt{9-x^2}$, $\lim_{x\to -3} \sqrt{9-x^2}$

(4) $\lim_{x\to 1} \dfrac{|1-x|}{1-x}$

(5) $\lim_{h\to 0} \dfrac{[(-1+h)^2+2]-3}{h}$

(6) $\lim_{x\to \infty} (x^2-x^3)$

(7) $\lim_{x\to 1^-} \dfrac{x}{1-x}$, $\lim_{x\to 1^+} \dfrac{x}{1-x}$, $\lim_{x\to 1} \dfrac{x}{1-x}$

(8) $\lim_{x\to \infty} \sqrt{x^2+1}-x$

(9)

Problem 2: Consider the function $f(x)=\dfrac{x-3}{x^2-x-6}$. Find:

(1) $\lim_{x\to 3} f(x)$

(2) $\lim_{x\to -2^-} f(x)$

(3) $\lim_{x\to -\infty} f(x)$

(4) the points of discontinuity of $f(x)$

Problem 3: Find all points of discontinuity of the function $f(x)=\dfrac{x^2-3x-10}{x^2-4}$ and identify the type of each one.

Problem 4: Consider the function $f(x)=3+\dfrac{1}{x-2}$.

(1) If exists, find $\lim_{x\to 0} f(x)$.

(2) If exists, find $\lim_{x\to 2} f(x)$.

(3) If exists, find $\lim_{x\to \infty} f(x)$.

(4) Find all values of x at which $f(x)$ is discontinuous.

(5) Find all vertical asymptotes, if any.

(6) Find all horizontal asymptotes, if any.

Problem 5: Find all values of *A, B,* and *C* which will make the following functions continuous.

(1) $f(x)=\begin{cases} C-x & \text{if } x\le 2, \\ x^2-C & \text{if } x>2. \end{cases}$

(2) $f(x)=\begin{cases} A-2x & \text{if } x\le 1 \\ B & \text{if } 1<x\le 2 \\ x^2-A & \text{if } x>2. \end{cases}$

DERIVATIVES

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(1) Use the definition of the derivative to find $f'(2)$ where $f(x)=1-x^2$.

(2) Use the definition of the derivative to find $f'(3)$ where $f(x)=\sqrt{x+1}$.

(3) Find the slope of the line tangent to the graph of $f(x)=x^2\sqrt{x}+\pi^2$ at $x=4$.

(4) Find the equation of the line tangent to the graph of $y=\dfrac{x}{1+\sqrt{x}}$ at $x=1$.

(5) Find the derivative at $x=0$ of the function: $f(x)=\left[\dfrac{1-x^2}{1+x^2}\right]^{-3}$

(6) Find the slope of the line tangent to the graph of the function $f(x)=\left[\dfrac{1-\sqrt{x}}{1+x^2}\right]^{-3}+\pi^2$ at $x=0$.

(7) The demand equation for a certain product is $p=200-\dfrac{1}{2}x$, when *x* units are sold at a price *p* dollars per unit. If the cost of producing x units is $C(x)=3000-\dfrac{1}{4}x^2$.

(a) Find the revenue and profit functions.
(b) Find the marginal revenue function.
(c) Approximate the cost of producing the unit number 11. **(Do not find the exact cost.)**

(8) The demand equation for a certain product is $p=\dfrac{108}{q+2}$, when *p* denotes the price per unit for *q* unit.

(a) Find the revenue and marginal revenue functions.
(b) Approximate the revenue from selling the unit number 11. **(Do not find the exact revenue.)**