

Serial No:

Student No.:

Name:

- 1. SHOW ALL WORK. NO CREDITS FOR ANSWERS NOT SUPPORTED BY WORK.**
2. CALCULATORS ARE NOT ALLOWED.

Problem 1 (25 Points): If the limit exists find it. If it does not exist, say so; use ∞ and $-\infty$ when appropriate.

(a) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 1}$

(b) $\lim_{x \rightarrow 0^+} \frac{x^2}{\sqrt{x^2 + 4} - 2}$

(c) $\lim_{x \rightarrow 3^+} \frac{-x^2}{9 - x^2}$

(d) $\lim_{x \rightarrow -\infty} \frac{2x^2 + 4x - 9}{1 + 3x - 3x^2}$

Problem 2 (25 Points)

- (a) Use the definition of the derivative to find $f'(2)$ for the function $f(x) = x^2 + x$.
- (b) Find the equation of the line tangent to the graph of $y = \sqrt{x} - \frac{2}{\sqrt{x}}$ at the point $(1, -1)$.
- (c) Find all points on the graph of $y = \frac{x^3}{3} - x^2$ where the slope is 3.

Problem 3 (25 Points)

- (a) Find all values of a and b which will make the function $f(x)$ continuous.

$$f(x) = \begin{cases} 2x + 1 & \text{if } x < 2 \\ b & \text{if } x = 2 \\ x^2 + a & \text{if } x > 2 \end{cases}$$

- (b) The demand equation of a certain product is $p = \frac{121}{q+2} + 3$, where p is the price per unit and q

denotes the number of units available. If the revenue function is $R(q) = pq$.

- (i) Find the marginal revenue at $q = 20$.
- (ii) Estimate the revenue from selling unit number 21.
- (iii) Find the relative rate of change in revenue when $q = 20$.

Problem 4 (25 Points):

(a) Find the derivative of $y = \frac{(x-2)(1+x^2)}{2x+1}$ at $x = 2$.

(b) If $y = 3u^3 - u^2 + 7u - 2$ and $u = 5x - 2$, find $\frac{dy}{dx}$ when $x = 1$.

(c) If $f(x) = \sqrt[5]{1 + (x^2 + 1)^3}$, find $f'(x)$. (Do Not Simplify the answer)