King Fahd University of Petroleum & Minerals

Math 101 - Sec. 5 & 10

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2^{nd} Major Exam	**Semester 051**	**Time: 90 min.**
Name:	**ID #:**	**Section #:**

Q1. (10 Points - Suggested time: 5 minutes) State if each of the following statements is true or false:

1. Every invertible function is 1-1.
2. If $f(x)$ is differentiable and $f(x) \geq 0$ for all $x \in \mathbb{R}$, then $\sqrt{f(x)}$ is differentiable.
3. $(\frac{d}{dx}\sin^{-1}(x))_{|_{x=\frac{1}{2}}} = \cos^{-1}(\frac{1}{2})$.
4. If $f(x)$ is not differentiable at $x = 0$, then $f^2(x)$ is not differentiable at $x = 0$.
5. If $f(x)$ and $g(x)$ are invertible, then $f(x)g(x)$ is invertible as well.

Q2. (10 Points - Suggested time: 10 minutes) Showing all details find

1. $\lim\limits_{x \mapsto \pi} \frac{\sin^3(x-\pi)}{(x-\pi)^3} =$

2. $\lim\limits_{x \mapsto 0}(x\cos(\frac{30}{x})) =$

Q3. (20 Points - Suggested time: 20 minutes) <u>Show all details</u>

1. Let $f(x) = ax^2+bx+c$ be such that the x-intercept is 1, the y-intercept is 3 and the tangent line to the curve of $f(x)$ at $x = 1$ is parallel to the line $2x + 3y - 2 = 0$. Find the constants a, b & c.

2. Find the point(s) at which the normal to the curve of

$$xy^2 - x + y = 0$$

is perpendicular to the line $y = x - 1$.

Q4. (20 Points - Suggested time: 15 minutes)

1. Showing all details, use the **definition** to show that if $f(x) = \sqrt{x}$, then $f'(x) = \frac{1}{2\sqrt{x}}$ for $x > 0$.

2. Let $f(1) = 1 = f'(1)$, $f(0) = 0$ and $f'(0) = \frac{3}{2}$. Assuming $F(x) = \sqrt[3]{f(x^3 + x + 1)} + f^2(x)$, find $F'(0)$.

Q5. (20 Points - Suggested time: 20 minutes) Show all details

1. Assume you have a cone, whose height is twice its radius. If the percentage error in measuring the radius of the cone is within $\pm$ 0.1 %. Estimate the percentage error in calculating the volume of that cone.

2. Using *linearization*, estimate $\tan(46°)$.

Q6. (20 Points - Suggested time: 20 minutes) Show all Details

1. Find the equation of the normal line to the curve of $y = \sec^2(\frac{x}{2})$ at $x = \frac{\pi}{2}$.

2. A closed right cylinder is known to have height equal to 3 times its radius. If (due to heat) the radius of the cylinder is increasing at a rate of 0.0001 cm/s, when its height is 12 *cm*. Find the rate at which the total surface area of the cylinder is increasing at that instant.

GOOD LUCK