

# King Fahd University of Petroleum & Minerals

## Math 101 - 14 & 18

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First Major Exam

Semester 041

Name:

ID #:

Section #:

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**Q1. (10 Points - Suggested time: 5 minutes)** State if each of the following statements is true or false:

1. If  $\lim_{t \rightarrow \infty} f(t) = 2$ , then  $\lim_{x \rightarrow 0} f(\frac{1}{x}) = 2$ .
2. If  $f(x)$  and  $g(x)$  are continuous at  $x = a$  then  $\frac{f(x)}{g(x)}$  is continuous at  $x = a$ .
3. The composition of any two continuous functions is continuous.
4. There exists a function  $f(x) : [0, 2\pi] \rightarrow \mathbb{R}$ , whose graph have infinite number of vertical asymptotes.
5. If  $\lim_{x \rightarrow a} f(x) = 0$  and  $\lim_{x \rightarrow a} g(x) = 0$  then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  does not exists.

**Q2. (20 Points - Suggested time: 15 minutes)** Consider the function:

$$f(x) := \begin{cases} \frac{x^3-1}{x-1}, & x < 3 \\ \sqrt{x} & i \neq j \end{cases}$$

1. Find the points at which the function is continuous.
2. Find the horizontal asymptotes of the function (if any)
3. Find the vertical asymptotes of the function (if any)

**Q3. (40 Points - Suggested time: 20 minutes)** Find the following limits, if they exist (Showing all details):

1.  $\lim_{x \rightarrow -\infty} \frac{2x^{777} - x^3 + 2}{x^3 - x^{777} + 1} =$

2.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 3x - 3} - x}{x - 1} =$

3.  $\lim_{x \rightarrow 1}$

$$4. \lim_{x \rightarrow 0} \frac{\sin(3x)}{4x} =$$

**Q4. (10 Points - Suggested time: 10 minutes)** Find the horizontal and the vertical asymptotes of

$$f(x) = \frac{x^4 - 6x^2 + 8x - 3}{x^4 + 4x^3 - 2x^2 - 12x + 9}.$$

**Q5. (10 Points - Suggested time: 10 minutes)** Show (using the definition) that

$$\lim_{x \rightarrow -1} (2x - 3) = -5$$