King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

Math 101 - 2 & 7

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CODE 1

Final Exam **Semester 031**
Maximum Time Allowed: 3 Hours

Name: **ID #:** **Section #:**

Q1. (10 Points) (Suggested time: 10 minutes) State if each of the following statements is TRUE or FALSE:

1. A relative minimum of a function may be larger than a relative maximum of that function.
2. A function $f(x)$ that is continuous on $[a, b]$ and differentiable on (a, b) has exactly one $c \in (a, b)$, such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.
3. A function $f(x)$ that is decreasing on (a, ∞) for some $a \in \mathbb{R}$ can have no absolute minimum.
4. Newton's Method can only converge to the right root.
5. $\log_b x$ is increasing for every $b > 0$ $(b \neq 1)$.

Q2. (10 Points) (Suggested time: 15 minutes) Find the area of the largest rectangle that can be inscribed in an equilateral triangle (with equal sides), the length of each of its sides is 8 inch.

CODE 1

Q3. (**60 Points**) (**Suggested time: 100 minutes**) **Encircle the most correct answer:**

1. $\lim_{x \mapsto 0} \frac{\tan(5x)}{\sin(3x)} =$
   (a) 0
   (b) $\frac{3}{5}$
   (c) $\frac{5}{3}$
   (d) $\frac{-5}{3}$
   (e) Does Not Exist

2. $\lim_{x \mapsto -1^-} \frac{x^2-5x-7}{x+1} =$
   (a) -7
   (b) $\frac{-11}{2}$
   (c) $-\infty$
   (d) ∞
   (e) none of the above

3. $\lim_{x \mapsto -\infty} (x + \ln(x^2 + 3)) =$
   (a) 0
   (b) ∞
   (c) $-\infty$
   (d) 1
   (e) none of the above

4. $f(x) = x^{\frac{2}{3}}$ has
   (a) discontinuity at $x_0 = 0$
   (b) a vertical tangent at $x_0 = 0$.
   (c) a cusp at $x_0 = 0$
   (d) a corner at $x_0 = 0$
   (e) none of the above

CODE 1

5. If $f(x) = \frac{1}{x^2-1}$ and $g(x) = \sqrt{1+x}$, then the domain of $(g \circ f)(x)$ is

   (a) $(-\infty, -1) \cup (1, \infty)$
   (b) $(-\infty, -1) \cup (1, \infty) \cup \{0\}$
   (c) $(-1, 1)$
   (d) $[-1, 1]$
   (e) none of the above

6. Let $f(x) = 2x - x^2$, $x \geq 1$. Then

   (a) $f^{-1}(x) = 1 - \sqrt{1-x} : (-\infty, 0] \longrightarrow [1, \infty)$.
   (b) $f^{-1}(x) = 1 - \sqrt{1-x} : (-\infty, 1] \longrightarrow [1, \infty)$.
   (c) $f^{-1}(x) = 1 + \sqrt{1-x} : (-\infty, 0] \longrightarrow [1, \infty)$.
   (d) $f^{-1}(x) = 1 + \sqrt{1-x} : (-\infty, 1] \longrightarrow [1, \infty)$.
   (e) none of the above

7. The graph of $f(x) = \frac{x^4-x^3-x+1}{x^3-x}$ has

   (a) oblique asymptote $y = x - 1$ and three vertical asymptotes $x = 0$, $x = 1$, $x = -1$.
   (b) oblique asymptote $y = x + 1$
   (c) horizontal asymptote $y = 1$
   (d) oblique asymptote $y = x - 1$ and two vertical asymptotes: $x = 0$, $x = -1$.
   (e) none of the above

8. The largest δ, such that

$$|x-2| < \delta \Rightarrow \left|x^2 - 4\right| < \epsilon$$

   is given by:

   (a) $\delta = \frac{\epsilon}{3}$
   (b) $\delta = \min\{1, \frac{\epsilon}{3}\}$
   (c) $\delta = \frac{\epsilon}{5}$
   (d) $\delta = \min\{1, \frac{\epsilon}{5}\}$
   (e) none of the above

9. The slope of the tangent to the graph of $x^2y^7 - x^3y^2 = 2$ at $(-1, 1)$ is:

   (a) $\frac{5}{9}$

   (b) $\frac{-5}{9}$

   (c) 1

   (d) -1

   (e) none of the above

10. The function $f(x) = \frac{-x}{x^2+1}$

    (a) has neither an absolute maximum nor an absolute minimum

    (b) has an absolute maximum but no absolute minimum

    (c) has an absolute minimum but no absolute maximum

    (d) has an absolute maximum and an absolute minimum

    (e) is not bounded

11. The set of values of c obtained by applying the Mean Value Theorem to $f(x) = \frac{1}{1-x}$ on $[3, 4]$ is:

    (a) Φ (i.e. there is no such c)

    (b) $\{1 + \sqrt{6}\}$

    (c) $\{1 - \sqrt{6}\}$

    (d) $\{1 + \sqrt{6}, 1 - \sqrt{6}\}$

    (e) none of the above

12. Applying Newton's Theorem to $f(x) = x^2 - 2$ with $x_1 = 1$, the value of x_3 will be

    (a) $\frac{17}{12}$

    (b) $\frac{9}{4}$

    (c) $-\frac{17}{12}$

    (d) $-\frac{9}{4}$

    (e) none of the above

13. If a particle is moving on the s-axis and its position versus time is given by $s(t) = t^3 - 3t^2 + 4$, $0 \leq t \leq 3$. Then the particle is slowing down on

   (a) $(0, 1)$
   (b) $(1, 2)$
   (c) $(2, 3)$
   (d) $(0, 1) \cup (2, 3)$
   (e) none of the above

14. If the equation $Q(t) = Q_0 e^{-kt}$ $(k > 0)$ gives the amount in grams of a radioactive element after t hours, then the time needed to reduce an amount of this element to half of its initial value is:

   (a) $\frac{\ln 2}{k}$ hours
   (b) $\frac{k}{\ln 2}$ hours
   (c) $\frac{\ln(2Q_0)}{k}$ hours
   (d) $\frac{\ln 2}{kQ_0}$ hours
   (e) none of the above

15. $\frac{d}{dx}((x^4 + 3)^{\cos x}) =$

   (a) $\cos x \cdot (x^4 + 3)^{\cos x - 1}$
   (b) $(x^4 + 3)^{\cos x} \ln(x^4 + 3)$
   (c) $4x^3(x^4 + 3)^{\cos x} \ln(x^4 + 3)$
   (d) $(x^4 + 3)^{\cos x} \cdot (\frac{4x^3 \cos(x)}{x^4+3} - \sin(x) \ln(x^4 + 3))$
   (e) none of the above

16. $\cos^{-1}(\cos(\frac{31\pi}{4})) =$

   (a) $\frac{31\pi}{4}$
   (b) $\frac{-\pi}{4}$
   (c) $\frac{\pi}{4}$
   (d) $\frac{3\pi}{4}$
   (e) none of the above

17. The instantaneous rate of change of $f(x) = \sec^{-1}(-(1+x^2))$, $|x| \geq 1$ is given by:

    (a) $\frac{-2x}{\sec(-1-x^2)\tan(-1-x^2)}$
    (b) $\frac{2x}{\sec(-1-x^2)\tan(-1-x^2)}$
    (c) $\frac{-1}{|x|(1+x^2)\sqrt{x^2+2}}$
    (d) $\frac{1}{|x|(1+x^2)\sqrt{x^2+2}}$
    (e) none of the above

18. The radius of a cylinder is measured with a percentage error within $\pm 0.08\%$, while its height is measured exactly. Then the percentage error in calculating the volume of the cylinder will then be within:

    (a) $\pm 0.0016\%$
    (b) $\pm 0.016\%$
    (c) $\pm 0.16\%$
    (d) $\pm 16\%$
    (e) none of the above

19. The Local Linear Approximation of $f(x) = \sin^{-1}(x)$ at $x_0 = \frac{-1}{\sqrt{2}}$ is:

    (a) $\sqrt{2}x + (1 - \frac{\pi}{4})$
    (b) $\sqrt{2}x + (\frac{\pi}{4} - 1)$
    (c) $-\sqrt{2}x + (1 - \frac{\pi}{4})$
    (d) $-\sqrt{2}x + (\frac{\pi}{4} - 1)$
    (e) none of the above

20. A square is inscribed in a circle, so that each of its corners lie on the circumference of the circle. If the radius of the circle increases at a rate of 2 $inch/hour$, when $r = 10$ $inch$, then the rate at which the area of the square is changing at that instant is

    (a) 80 $inch^2/hour$
    (b) 8 $inch^2/hour$
    (c) $20\sqrt{2}$ $inch^2/hour$
    (d) $2\sqrt{2}$ $inch^2/hour$
    (e) none of the above

CODE 1

Q5. (**20 Points**) (**Suggested time: 25 minutes**) Consider the function:

$$f(x) = -x^2 e^{-x}$$

1. **Find each of the following: (Details should be included on the back).**

   (a) Domain($f(x)$) =

   (b) Range($f(x)$) =

   (c) x-intercept(s) (if any):

   (d) y-intercept:

   (e) Symmetries (if any):

   (f) $\lim_{x \mapsto \infty} f(x) =$

   (g) $\lim_{x \mapsto -\infty} f(x) =$

   (h) Asymptotes (if any):

   (i) Critical Points (if any):

   (j) Interval(s) on which $f(x)$ is increasing (if any):

   (k) Interval(s) on which $f(x)$ is decreasing (if any):

   (l) Relative Maximum (if any):

   (m) Relative Minimum (if any):

   (n) Absolute Maximum (if any):

   (o) Absolute Minimum (if any):

   (p) Intervals on which the graph of $f(x)$ is concave up (if any):

   (q) Intervals on which the graph of $f(x)$ is concave down (if any):

   (r) Inflection Points (if any):

2. **Draw the graph of $f(x)$ (The final graph should be included on this page)**