

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DEPARTMENT OF MATHEMATICAL SCIENCES

MATH 551 (ABSTRACT ALGEBRA)

SEMESTER 072 (SPRING 2008)

DR. JAWAD ABUIHLAIL

1) **Description:** *Basic Definitions of Rings and Modules, Homomorphisms, Sums and Products, Exactness, Hom and Tensor, Adjoint Isomorphisms, Free, Projective and Injective Modules. Chain Conditions, Primary Decomposition, Noetherian Rings and Modules, Artinian Rings, Structure theorems.*

2) **Prerequisite:** Math 345 (Math 450 is recommended)

3) **TEXTBOOKS:**

- P. Grillet, *Abstract Algebra*, 2nd edition, Springer (2007).
- S. Lang, *Algebra*, Revised 3rd edition, Springer (2005).

4) **FURTHER READING:**

- N. Bourbaki, *Elements of mathematics: Algebra I*, Springer (1988).
- D.S. Dummit and R.M. Foote, *Abstract Algebra*, 3rd edition, John Wiley & Sons (2004).
- T. Hungerford, *Algebra*, Graduate Texts in Mathematics 73, Springer-Verlag, New York-Berlin (1980).

5) **GRADING POLICY:**

| | |
|------------------|------------|
| Exam I | 200 |
| Exam II | 200 |
| Research Project | 100 |
| Total | 500 |

Detailed Syllabus¹

| Week(s) | Section(s) | Title | Highlights |
|-------------------------------------------------|---------------------|----------------------------------------------------|-----------------------------------------------------------------------------|
| Chapter III. Rings | | | |
| 1-2 | III.1-III.2 | Rings, Subrings and Ideals | Definitions & examples |
| | III.3 | Homomorphisms | Theorem 3.6. |
| | III.4 | Domains and Fields | 4.5. – 4.7., 4.11. |
| 3- 4 | III.5 & III.6 | Polynomials in One & Several Variables | 5.1., 5.5., 5.7., 5.11., 5.12.; 6.8. |
| | III.7* ² | Formal Power Series | 7.3., 7.5. |
| | III.8 | Principal Ideal Domains | 8.3., 8.4., 8.11., 8.12. |
| 5 | III.10 | Unique Factorization Domains | 10.2., 10.4., 10.9. |
| | III.11 | Noetherian Rings | 11.1., 11.2. (Hilbert's Basis Theorem) |
| Chapter VII. Commutative Rings | | | |
| 6 | VII.1. | Primary Decomposition | 1.9., 1.10. (Noether-Lasker) |
| | VII.4 | Localization | 4.2., 4.5., 4.7., 4.10. |
| Chapter VIII. Modules | | | |
| 7 | VIII.1. | Definition | Definitions and examples |
| | VIII.2. | Homomorphisms | Theorems 2.6. – 2.8. |
| 8 | III.3. | Direct Sums and Products | 3.1., 3.2., 3.5. |
| | III.4 & III.5 | Free Modules / Vector Spaces | 4.2., 4.5., 4.6. / 5.3.; 5.5. |
| 9 | III.6 | Modules over Principal Ideal Domains | 6.1. – 6.3. |
| 10 | III.8 | Chain Conditions (Modules of Finite Length) | Equivalent conditions defining Noetherian/Artinian Modules 8.8., 8.10 |
| Chapter IX. Semisimple Rings and Modules | | | |
| 11 | IX.1 | Simple Rings and Modules | 1.2.(Schur's Lemma), 1.8., 1.9. |
| | IX.2 | Semisimple Modules | 2.1. |
| | IX.3 | The Artin Wedderburn Theorem | 3.1., 3.3., 3.8. |
| 12 | IX.4* | Primitive Rings | Jacobson Density Theorem |
| | IX.5 | The Jacobson Radical | 5.1., 5.2., 5.6., 5.6. & 5.7. (Nakayama's Lemma) |
| | IX.6 | Artinian Rings | 6.1. – 6.4. (Hopkins-Levitzki) |
| Chapter X. Projectives and Injectives | | | |
| 13 | X.1 | Exact Sequences | Five-Lemma, Nine-Lemma |
| | X.2* | Pullbacks and Pushouts | Existence/Uniqueness & Basic Properties |
| 14 | X.3 | Projective Modules | 3.2., 3.6., 3.7. |
| | X.4 | Injective Modules | 4.1., 4.5. (Baer's Criterion), 4.7., 4.11., 4.12. |
| | X.5* | The Injective Hull | 5.6. (Existence & Uniqueness) |
| | X.6* | Hereditary Rings | Definition, Dedekind Domains |
| Chapter XI. Construction | | | |
| 15-16 | XI.1 | Groups of Homomorphisms | 1.4., 1.5. |
| | XI.2 | Properties of Hom | 2.1., 2.5. |
| | XI.5 & XI.6 | Tensor Products and their Properties | Existence/Uniqueness; 6.6., 6.7., 6.9. |
| | XI.7* | Dual Modules | 7.4., 7.5., 7.7. |
| | XI.8 | Flat Modules | Basic Properties, 8.7. 8.11. |

Numbers refer to Grillet's book "Abstract Algebra"¹
Sections marked with "*" will be covered as projects by students²