

**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS**  
**DEPARTMENT OF MATHEMATICAL SCIENCES**

**MATH 553 ( HOMOLOGICAL ALGEBRA)**  
**SPRING 2007 (SEM. 062)**

**DR. JAWAD ABUHLAIL**

**DESCRIPTION:** Review of free, projective, and injective modules, direct limits. Watt's theorems. Flat modules. Localization. Noetherian, semisimple, Von Neumann regular, hereditary, and semi-hereditary rings. Homology, homology functors, derived functors. Ext. and Tor. homological dimensions, Hilbert syzygy theorem.

**Prerequisite:** MATH 551

**TEXT BOOK:** M. Osborne, *Basic Homological Algebra*, Springer (2000).  
accompanied by: J. Rotman, *An Introduction to Homological Algebra*, Academic Press (1979).

**MORE BOOKS IN HOMOLOGICAL ALGEBRA:**

- 1) H. Cartan and S. Eilenberg, *Homological Algebra*, Princeton University Press (1956).
- 2) E. Enochs & O. Jenda, *Relative Homological Algebra*, Gruyter Expositions in Mathematics: Vol. 30 (2000).
- 3) P. Hilton and U. Stambach, *A Course in Homological Algebra*, Springer-Verlag (1971).
- 4) D. Northcott, *An Introduction to Homological Algebra*, Cambridge University Press (1960).
- 5) C. Weibel, *An Introduction to Homological Algebra*, Cambridge University Press, Cambridge (1994).

**FURTHER READING:**

- 1) L. Fuchs & L. Salce, *Modules over Non-Noetherian Domains*, AMS (2000).
- 2) S. Lang, *Algebra (revised 3<sup>rd</sup> edition)*, Springer (2002).
- 3) H. Matsumura, *Commutative Ring Theory*. Cambridge University Press (2004).
- 4) R. Wisbauer, *Foundations of module and ring theory, A handbook for study and research*, Gordon and Breach Science Publishers (1991).

**GRADING POLICY:**

Take-home Exam I	200 Points
Take-home Exam II	200 Points
Project	100 Points

## SYLLABUS

Chapter	Sec.	Core Material	Week(s)
<b>1) Categories</b> (including selected parts of Chapter 7)	<b>1.1.</b>	<i>Categories and Functors</i> (including parts of 7.1, 7.2 & 7.3)	<b>1</b>
<b>2) Modules</b> (including selected parts of Chapters 7, 8 and 9)	<b>2.1.</b>	<i>Generalities</i> (including <i>Limits and Colimits</i> 8.1.)	<b>2</b>
	<b>2.2.</b>	<i>Tensor Products</i>	
	<b>2.3.</b>	<i>Exactness of Functors</i>	<b>3</b>
	<b>2.4.</b>	<i>Review of Projectives, Injectives, and Flats</i> (including <i>Injective Envelopes</i> 9.1.) - (semi)hereditary rings - Von Neumann regular rings	
		<i>Watt's Theorems</i> ( <b>Rotman</b> pp. 75-81)	<b>4</b>
<b>3) Ext and Tor</b> (including selected parts of Chapters 6 and 7)	<b>3.1.</b>	<i>Complexes and Projective Resolutions</i>	<b>5</b>
	<b>3.2.</b>	<i>Long Exact Sequences</i> (including parts of 6.3, 6.4, 6.5 & 7.7)	<b>6</b>
	<b>3.3.</b>	<i>Flat Resolutions and Injective Resolutions</i>	<b>7</b>
	<b>3.4.</b>	<i>Consequences</i>	
<b>First Take-Home Exam</b>			
<b>4) Dimension Theory</b>	<b>4.1.</b>	<i>Dimension Shifting</i>	<b>8</b>
	<b>4.2.</b>	<i>When Flats are Projectives</i>	
	<b>4.3.</b>	<i>Dimension Zero (semisimple rings)</i>	<b>9-10</b> (3 lectures)
	<b>4.4.</b>	<i>An Example</i>	
<b>5) Change of Rings</b>	<b>5.1.</b>	<i>Computational Considerations</i>	<b>11</b>
	<b>5.2.</b>	<i>Matrix Rings</i>	
	<b>5.3.</b>	<i>Polynomials (Hilbert's Syzygy's Theorem &amp; Serre's Theorem)</i>	<b>12</b>
	<b>5.4.</b>	<i>Quotients and Localization</i>	<b>13</b>
<b>Second Take-Home Exam</b>			
<b>6) Derived Functors</b>	<b>6.1.</b>	<i>Additive Functors</i>	<b>14</b>
	<b>6.2.</b>	<i>Derived Functors</i>	
		<i>Selected parts of 6.3-6.6 will be embedded in pervious sections</i>	
<b>7) Abstract Homological Algebra</b>		<i>Selected parts of 7.1 – 7.3 &amp; 7.6 - 7.7 will be embedded in previous chapters</i>	
<b>Catching Up/ Presentations</b>		<i>Homological Lemmas</i> (selected books & research papers)	<b>15-16</b> (3 lectures)