

King Fahd University of Petroleum & Minerals

Math 101 - 14 & 18

Dr. Jawad Y. Abuhlail

3rd Major Exam

Semester 042

Time: 90 min.

Name:

ID #:

Section #:

Q1. (10 Points - Suggested time: 5 minutes) State if each of the following statements is true or false:

1. Every function with inverse is monotone.
2. $\cos^{-1}(\cos \frac{3\pi}{2}) = \frac{3\pi}{2}$.
3. $\frac{d}{dx} \tan^{-1}(x^2 + 1) = \frac{1}{1+(x^2+1)^2}$.
4. $\cos^{-1}(x) = \sec^{-1}(\frac{1}{x})$ for all $x \in [-1, 1] \setminus \{0\}$.
5. $\log_a x = \frac{\ln x}{\log a}$ for all $x > 0$ and $a > 0, a \neq 1$.

Q2. (15 Points - Suggested time: 10 minutes) Graph the curve of $f(x) = 2 + \ln(x + 1)$ (showing all possible steps).

Q3. (15 Points - Suggested time: 15 minutes) State whether each of the following functions has an inverse or not (justify your answer).

1. $f(x) = \cos(x) + 3x, x \in \mathbb{R}$

2. $f(x) = x^2 - x + 1, x \in \mathbb{R}$

3. $f(x) = \frac{1}{x+1}, x \neq -1$

Q4. (10 Points - Suggested time: 10 minutes) Show that

$$\frac{d}{dx} \sec^{-1}(x) = \frac{-1}{x\sqrt{x^2-1}} \text{ for } x < -1.$$

Q5. (15 Points - Suggested time: 15 minutes) Find the following limits (if they exist)

1. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{4x^2}\right)^{2x}$

2. $\lim_{x \rightarrow 0^+} x \ln\left(\frac{1}{x}\right)$

3. $\lim_{x \rightarrow 0^-} \frac{x \sin\left(\frac{1}{x}\right)}{\tan x}$

Q6. (20 Points - Suggested time: 20 minutes) Find:

1. The equation of the tangent to the curve of $f(x) = x \sin^{-1}(x)$ at $x = \frac{1}{2}$.

2. The equation of the normal to the curve of $f(x) = \ln(xe^x)$ at $x = e$.

Q7. (10 Points - Suggested time: 10 minutes) Find all possible values of k and l so that

$$\lim_{x \rightarrow 0} \frac{\cos(lx) - k}{x^2} = -2.$$

Q8. (5 Points - Suggested time: 5 minutes) Solve the equation

$$\frac{e^x - e^{-x}}{2} = 2.$$

GOOD LUCK