King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

MATH-533: Complex Variables I
Spring Semester 2004 (032)

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Second Major - TakeHome

Name: **ID:**

Q1. (15 Points) Show that for $n = 2, 3, 4, 5, ...$

$$S_n := \sin \frac{\pi}{2} \cdot \sin \frac{2\pi}{n} \cdot ... \cdot \sin \frac{(n-2)\pi}{n} \cdot \sin \frac{(n-1)\pi}{n} = \frac{n}{2^{n-1}}.$$

Q2. (15 Points) Find a linear fractional transformation which carries

$$C_1 := \{z \in \mathbb{C} : |z| = 1\} \text{ and } C_2 := \{z \in \mathbb{C} : \left|z - \frac{1}{4}\right| = \frac{1}{4}\}$$

into cocentric circles. What is the ratio of the radii?

Q3. Let $f : \Omega \to \mathbb{C}$ be such that the differential of f exists and is different from 0 at $z_0 \in \Omega$. Show that f is conformal at z_0 if and only if

$$\lim_{r \mapsto 0} e^{-i\theta} \frac{f(z_0 + re^{i\theta}) - f(z_0)}{|f(z_0 + re^{i\theta}) - f(z_0)|}, \ r > 0$$

exists and is independent of θ.

Q4.

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