

King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

MATH-533: Complex Variables I
Spring Semester 2004 (032)

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Homework # 5

Due: Monday 10.5.2004

Q1. Find all circles which are orthogonal to $|z| = 1$ and $|z - 1| = 4$.

Q2. Map the inside of the right-hand branch of the hyperbola $x^2 - y^2 = a^2$ on the disk $|w| < 1$, so that the focus corresponds to $w = 0$ and the vertex corresponds to $w = -1$.

Q3. Assume that $f(z)$ is analytic and satisfies $|f(z) - 1| < 1$ in a region Ω . Show that $\int_{\gamma} \frac{f'(z)}{f(z)} dz = 0$ for every closed curve in Ω .

Q4. Compute

$$\int_{|z|=\rho} \frac{|dz|}{|z - a|^2}, \quad |a| \neq \rho.$$

Q5. Show that the successive derivative of an analytic function $f(z)$ can never satisfy $|f^{(n)}(z)| > n!n^n$.

Q6. Show that a function which is analytic in the whole plane and has nonessential singularity at ∞ reduces to a polynomial.

Q7. Let $f(z)$ be analytic at the origin with $f'(0) \neq 0$. Prove that there exists an analytic function $g(z)$ such that $f(z^n) = f(0) + g(z)^n$ is a neighborhood of 0.

GOOD LUCK