

King Fahd University of Petroleum & Minerals  
Department of Mathematical Sciences

MATH-533: Complex Variables I  
Spring Semester 2004 (032)

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**Homework # 4**

Due: Saturday 17.4.2004

**Q1.** Give a precise definition of a single-valued branch of  $f(z) = \log(\log(z))$  in a suitable region and prove that it's analytic.

**Q2.** Suppose that  $f(z)$  is analytic with  $|f(z)^2 - 1| < 1$  in a region  $\Omega$ . Show that  $\operatorname{Re}(f(z)) > 0$  or  $\operatorname{Re}(f(z)) < 0$  throughout  $\Omega$ .

**Q3.** Show that any linear fractional transformation which transforms the real axis into itself can be written with real coefficients.

**Q4.** Show that any four distinct points can be carried by a linear fractional transformation to positions  $1, -1, k, -k$ , where the value of  $k$  depends on the points. How many solutions are there and how are they related?

**GOOD LUCK**