King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

MATH-533: Complex Variables I
Spring Semester 2004 (032)

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Third Major

Name: **ID:**

Q1. (10 Points - Suggested time: 10 minutes). State if each of the following statements is true or false:

1. The number of zeros of $P(z) = z^{87} + 36z^{57} + 71z^4 + z^3 - z + 1$ inside the unit circle is 3 roots.
2. $\oint\limits_{|z|=1} |z-1|\,|dz| = 8$.
3. $\left| \oint\limits_{|z-1|=1} \frac{e^z}{z+3} dz \right| \leq 2\pi e^2$.
4. $f(z) = \frac{e^z - 1}{z}$ has a simple pole at $z_0 = 0$.
5. Let $p(z)$ and $q(z)$ be analytic functions with $p(z_0) \neq 0$ and $q(z_0) = 0$. If $q'(z_0) \neq 0$, then $f(z) := \frac{p(z)}{q(z)}$ has a simple pole at z_0 with $\operatorname*{Res}_{z=z_0} f(z) = \frac{p(z_0)}{q'(z_0)}$.

Q2. (20 Points - Suggested time: 20 minutes) Give a counter examples to each of the following <u>false</u> statements:

1. If $f(z) : \Omega \to \mathbb{C}$ is a function with $\int\limits_C f(z)dz = 0$ (Ω is an open disk and $C \subset \Omega$ is a circle), then $f(z)$ is analytic in Ω.

2. Every simply connected region $\Omega \subset \mathbb{C}$ is connected.

Q3. (40 Points - Suggested time: 40 minutes). Prove any four of the following statements:

1. Let $f(z)$ be a function analytic inside and on the unit circle. Suppose that $|f(z) - z| < |z|$ on the unit circle. Show that $\left|f'(\frac{1}{2})\right| \leq 8$ and that $f(z)$ has precisely one zero inside the unit circle.

2. Every polynomial $P(z) \in \mathbb{C}[z]$ with degree $n \geq 1$ has at least one complex root.

3. If $f(z)$ is analytic in a rectangular region R (defined by $a \leq x \leq b$ and $c \leq y \leq d$), then $\int_{\partial R} f(z)dz = 0$.

4. $\int\limits_{\partial D} \frac{1}{1+z^{2n}} dz = \frac{\pi}{n \sin(\frac{\pi}{2n})}$ (for $n = 1, 2, 3, ...$), where $D = \{z \in \mathbb{C} : |z| < 2 \text{ and } \mathrm{Im}(z) > 0\}$.

5. Let $p(z)$ and $q(z)$ be analytic functions with $p(z_0) \neq 0$ and $q(z_0) = 0$. Show that z_0 is a zero of $q(z_0)$ with order h if and only if $f(z) := \frac{p(z)}{q(z)}$ has a pole with order h at z_0.

Q4. (30 Points - Suggested time: 30 minutes). Evaluate each of the following integrals:

1. $\oint\limits_{|z|=4} \frac{z^2-\pi z}{\sin z} dz$

2. $\oint_{|z|=2} \frac{2\sin(z^3)}{(z-1)^4} dz$

3. $\int_{-\infty}^{\infty} \frac{x^2+x+1}{x^4+1} dx$

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