King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

MATH-533: Complex Variables I
Spring Semester 2004 (032)

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Second Major: Take-Home

Name: **ID:**

Solve the first 4, **or** any 5 of the following problems:

Q1. Show that for $n = 2, 3, 4, 5, ...$

$$S_n := \sin\frac{\pi}{n} \cdot \sin\frac{2\pi}{n} \cdot ... \cdot \sin\frac{(n-2)\pi}{n} \cdot \sin\frac{(n-1)\pi}{n} = \frac{n}{2^{n-1}}.$$

Q2. Let $\Omega \subseteq \mathbb{C}$ be a region and $f : \Omega \to \mathbb{C}$ be such that the differential of f exists and is different from 0 at $z_0 \in \Omega$. Show that f is conformal at z_0 if and only if

$$\lim_{r \mapsto 0} e^{-i\theta} \frac{f(z_0 + re^{i\theta}) - f(z_0)}{|f(z_0 + re^{i\theta}) - f(z_0)|}, \ r > 0$$

exists and is independent of θ.

Q3. Consider the linear fractional transformation

$$f(z) = \frac{z - i}{z + i}.$$

What is the image of the real line $\mathbb{R}$ (respectively $\mathbb{R} \cup \{\infty\}$) under the map $w := f(z)$?

Q4. Find a linear fractional transformation which carries

$$C_1 := \{z \in \mathbb{C} : |z| = 1\} \text{ and } C_2 := \{z \in \mathbb{C} : \left|z - \frac{1}{4}\right| = \frac{1}{4}\}$$

into cocentric circles. What is the ratio of the radii?

Q5. Find a conformal mapping that takes the half plane on and to the left of the line $y = mx$ $(m > 0)$ onto the unit disk.

Q6. Show that any conformal mapping of the unit disk onto itself is of the form

$$h(z) = e^{i\theta} \frac{z - \beta}{1 - \overline{\beta} z}, \ \ |\beta| < 1.$$

GOOD LUCK