King Fahd University of Petroleum & Minerals

Math 102 - 22

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First Major Exam **Semester 032**

Name: **ID #:** **Section #:**

Q1. (10 Points - Suggested time: 5 minutes) State if each of the following statements is true or false:

1. Every integrable function over a closed interval $[a, b]$ is continuous.
2. If $f + g$ is integrable on an interval I, then f and g are integrable on I.
3. If F and G are antiderivatives of $f(x)$ over $[a, b]$, then $F(b) - G(b) = F(a) - G(a)$.
4. $\lim\limits_{n\mapsto\infty}(\frac{1}{n^2}\sum_{k=1}^{n-1} k) = \frac{1}{2}$
5. For any continuous function $f(x)$ over $[a, b]$ we have $\left|\int f(x)dx\right| = \int |f(x)|\, dx$.

Q2. (10 Points - Suggested time: 15 minutes) Use the **rectangular method** (with the right-hand approximation) to find the Net Signed Area (NSA) between the graph of $f(x) = x^2 - x$ and the x-axis over $[-1, 1]$.

Q3. (10 Points - Suggested time: 10 minutes). Find the area between the graph of $f(x) = \frac{1}{2} - \sin(x)$ and the x-axis over the interval $[0, \frac{\pi}{2}]$.

Q4. (10 Points - Suggested time: 10 minutes). Find the value(s) of c satisfying the Mean Value Theorem for Integration for $f(x) = \sqrt{x-1}$ over $[1, 5]$.

Bonus: (5 Points) Show that

$$\lim_{n \mapsto \infty} \sum_{k=1}^{n} \frac{1}{2n+k} = \ln(\frac{3}{2})$$

Q5. (60 Points - Suggested time: 40 minutes) Evaluate the following (**showing all details**).

1. $\int \frac{1}{x\ln(x)}dx$ $(x > 0)$.

2. $\int \frac{e^{3x}-e^x}{e^x+1}dx$

3. $\frac{d}{dx}(\int_1^{\cos(x)} \frac{\ln(|t|)}{t^2+1}dt)$

4. $\int \cos^3(x)dx$

5. $\int_{-2}^{2} \sqrt{16-4x^2}dx$

6. $\int_0^{\frac{5}{3}} \frac{1}{9x^2+25}dx$

GOOD LUCK