King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

Math 101 - 2 & 7

Dr. Jawad Y. Abuihlail

First Major Exam **Semester 031**
Time: 17:15-18:45 pm, Monday 13.10.2003

Name: **ID #:** **Section #:**

Q1. (10 Points) (Suggested time: 10 minutes) State if each of the following statements is TRUE or FALSE:

1. The function $f(x) = \frac{x^2-1}{x^2+x-2}$ has a removable discontinuity at $x = 1$.
2. If $f(x)$ and $\frac{f(x)}{g(x)}$ are continuous at $x = c$, then $g(x)$ is also continuous at $x = c$.
3. If $f(x)$ and $g(x)$ are defined everywhere and continuous at $x = c$, then $f \circ g$ is also continuous at $x = c$.
4. There is no $c \in \mathbb{R}$ that makes $f(x)$ continuous everywhere:
$$f(x) = \begin{cases} \frac{4-x^2}{x-2}, & x < 2 \\ cx^2 & x \geq 2 \end{cases}$$
5. The equation $x^3 - x - 2 = 0$ has at least one solution in $[-1, 2]$.

Q2. (48 Points) (Suggested time: 30 minutes) Find the following limits (if they exist). Show all details:

1. $\lim\limits_{x \mapsto -2} \frac{2+x}{x^3+8}$

2. $\lim\limits_{x\mapsto -1^-} \frac{1}{[x^2-2]}$

3. $\lim\limits_{x\mapsto 0}(2x \cdot \csc(3x))$

4. $\lim\limits_{x\mapsto \infty} \frac{2+x^2-x^3}{1+2x^2+x^5}$

5. $\lim_{x \mapsto -\infty} \frac{\left|x^3-3x+2\right|}{-5x^3+2}$

6. $\lim_{x \mapsto -\infty} \frac{\sqrt{x^2-2x+1}}{3-x}$

Q3. (**5 Points**) (**Suggested time: 10 minutes**) Show that there exists some $c \in [0, \frac{\pi}{2}]$, such that $\frac{\sin(c)}{c} = \frac{8}{10}$.

Q4. **(15 Points)** (**Suggested time: 10 minutes**) Draw a graph of a function $f(x)$ satisfying the following properties:

1. $f(-2) = -3$.
2. $f(0) = 0$.
3. $\lim_{x \mapsto 0} f(x) = 1$.
4. $f(x)$ has vertical asymptotes $x = 1$ and $x = -1$.
5. $f(x)$ has horizontal asymptotes $y = 2$ and $y = -2$.
6. $\lim_{x \mapsto 1^-} f(x) = 3$.
7. $\lim_{x \mapsto -1^+} f(x) = 1$.

Q5. (**10 Points**) (**Suggested time: 10 minutes**) Use definition, to show that:

1. $\lim_{x \mapsto \infty} \sqrt{x^2 - 1} = \infty$.

2. $f(x) = x^3 + 1$ is continuous at $x = 1$.

Q6. (12 Points) (Suggested time: 10 minutes) Give a *counter example* to each of the following **false** statements:

1. If $f(x)$ is continuous on $[a, b]$ and k is a real number between $f(a)$ and $f(b)$, then there exists exactly one $c \in [a, b]$ with $f(c) = k$.

2. If $f(x)$ and $g(x)$ are defined on (a, ∞) for some real number a with $\lim_{x \mapsto \infty} f(x) = \infty$ and $\lim_{x \mapsto \infty} g(x) = \infty$, then $\lim_{x \mapsto \infty} (f - g)(x) = 0$.

GOOD LUCK