# King Fahd University of Petroleum & Minerals
# Department of Mathematical Sciences

## Math 101 - 2 & 7

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**Third Major Exam** **Semester 031**
**Maximum Allowed Time: 3 Hours**

**Name:** **ID #:** **Section #:**

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**Q1. (10 Points) (Suggested time: 10 minutes)** State if each of the following statements is TRUE or FALSE:

1. A rational function may have horizontal and oblique asymptotes.
2. A function $f(x)$ that is continuous on $[a,b]$ and differentiable on $(a,b)$ has exactly one $c \in (a,b)$, such that $f(c) = \frac{f(b)-f(a)}{b-a}$.
3. A function $f(x)$ that is decreasing on $(a,\infty)$ for some $a \in \mathbb{R}$ can have no absolute minimum.
4. Newton's Method can only converge to the right root.
5. $\log_b x$ is increasing for every $b > 0$ $(b \neq 1)$.

**Q2. (10 Points) (Suggested time: 15 minutes)** Find the area of the largest rectangle that can be inscribed in an equilateral triangle (with equal sides), the length of each of its sides is 8 inch.

**Q3.** (**60 Points**) (**Suggested time: 75 minutes**) **Encircle the most correct answer:**

1. $\lim_{x \mapsto 0} \frac{\tan(5x)}{\sin(3x)} =$

   (a) $\frac{-3}{5}$
   (b) $\frac{3}{5}$
   (c) $\frac{5}{3}$
   (d) $\frac{-5}{3}$
   (e) Does Not Exist

2. $\lim_{x \mapsto -1^-} \frac{x^2-5x-7}{x+1} =$

   (a) $-7$
   (b) $\frac{-11}{2}$
   (c) $-\infty$
   (d) $\infty$
   (e) None of the above

3. $\lim_{x \mapsto -\infty} (x + \ln(x^2+3)) =$

   (a) 0
   (b) $\infty$
   (c) $-\infty$
   (d) 1
   (e) none of the above

4. $f(x) = x^{\frac{2}{3}}$ has

   (a) discontinuity at $x_0 = 0$
   (b) a vertical tangent at $x_0 = 0$.
   (c) a cusp at $x_0 = 0$
   (d) a corner at $x_0 = 0$
   (e) none of the above

5. If $f(x) = \frac{1}{x^2-1}$ and $g(x) = \sqrt{1+x}$, then the domain of $(g \circ f)(x)$ is

   (a) $(-\infty, -1] \cup [1, \infty)$
   (b) $(-\infty, -1) \cup (1, \infty)$
   (c) $(-1, 1)$
   (d) $[-1, 1]$
   (e) none of the above

6. Let $f(x) = 2x - x^2$, $x \geq 1$. Then

   (a) $f^{-1}(x) = 1 - \sqrt{1-x} : (-\infty, 0] \longrightarrow [1, \infty)$.
   (b) $f^{-1}(x) = 1 - \sqrt{1-x} : (-\infty, 1] \longrightarrow [1, \infty)$.
   (c) $f^{-1}(x) = 1 + \sqrt{1-x} : (-\infty, 0] \longrightarrow [1, \infty)$.
   (d) $f^{-1}(x) = 1 + \sqrt{1-x} : (-\infty, 1] \longrightarrow [1, \infty)$.
   (e) none of the above

7. The graph of $f(x) = \frac{x^4-x^3-x+1}{x^3-x}$ has

   (a) oblique asymptote $y = x - 1$ and three vertical asymptotes $x = 0$, $x = 1$, $x = -1$.
   (b) oblique asymptote $y = x + 1$
   (c) horizontal asymptote $y = 1$
   (d) oblique asymptote $y = x - 1$ and two vertical asymptotes: $x = 0$, $x = -1$.
   (e) none of the above

8. The largest $\delta$, such that

$$|x - 2| < \delta \Rightarrow \left|x^2 - 4\right| < \epsilon$$

   is given by:

   (a) $\delta = 1$
   (b) $\delta = 2$
   (c) $\delta = \frac{\epsilon}{3}$
   (d) $\delta = \min\{1, \frac{\epsilon}{3}\}$
   (e) none of the above

9. The slope of the tangent to the graph of $x^2y^7 - x^3y^2 = 2$ at $(-1, 1)$ is:

   (a) $\frac{5}{9}$
   (b) $\frac{-5}{9}$
   (c) $\frac{-9}{5}$
   (d) $\frac{9}{5}$
   (e) none of the above

10. The function $f(x) = \frac{-x}{x^2+1}$

   (a) has neither an absolute maximum nor an absolute minimum
   (b) has an absolute maximum but no absolute minimum
   (c) has an absolute minimum but no absolute maximum
   (d) has an absolute maximum and an absolute minimum
   (e) is not bounded

11. The set of values of $c$ obtained by applying the Mean Value Theorem to $f(x) = \frac{x+1}{2-x}$ on $[3, 4]$ is:

   (a) $\Phi$ (i.e. there is no such $c$)
   (b) $\{2 + \sqrt{2}\}$
   (c) $\{2 - \sqrt{2}\}$
   (d) $\{2 + \sqrt{2}, 2 - \sqrt{2}\}$
   (e) none of the above

12. In applying Newton's Theorem to $f(x) = x^2 - 2$ with $x_1 = 1$, the value of $x_3$ will be

   (a) $\frac{17}{12}$
   (b) $\frac{1}{4}$
   (c) $-\frac{17}{12}$
   (d) $-\frac{1}{4}$
   (e) none of the above

13. If a particle is moving on the $s$-axis and its position versus time is given by $s(t) = t^3 - 3t^2 + 4$, $0 \leq t \leq 3$. Then the particle is slowing down on

    (a) $(0, 1)$
    (b) $(1, 2)$
    (c) $(2, 3)$
    (d) $(0, 1) \cup (2, 3)$
    (e) none of the above

14. If the equation $Q(t) = Q_0 e^{-kt}$ $(0 < k < 1)$ gives the amount in grams of a radioactive element after $t$ hours, then the time needed to reduce the amount of this element to half of its initial value is:

    (a) $\frac{\ln 2}{k}$ hours
    (b) $\frac{k}{\ln 2}$ hours
    (c) $\frac{\ln(2Q_0)}{k}$ hours
    (d) $\frac{\ln 2}{kQ_0}$ hours
    (e) none of the above

15. $\frac{d}{dx}((x^4 + 3)^{\cos x}) =$

    (a) $\cos x \cdot (x^4 + 3)^{\cos x - 1}$
    (b) $(x^4 + 3)^{\cos x} \ln(x^4 + 3)$
    (c) $(x^4 + 3)^{\cos x} \cdot (\frac{4x^3 \cos(x)}{x^4+3} - \sin(x) \ln(x^4 + 3))$
    (d) $\frac{4x^3 \cos(x)}{x^4+3} - \sin(x) \ln(x^4 + 3)$
    (e) none of the above

**Q5.** (**10 Points**) (**Suggested time: 30 minutes**) Consider the function:

$$f(x) = \frac{x-2}{x^2-1}.$$

| | | | |
|---|---|---|---|
| 1) | the domain | 2) | the range |
| 3) | the $x$-intercept(s) | 4) | the $y$-intercept |
| 5) | the symmetries (if any) | 6) | the asymptotes (if any) |
| 7) | intervals on which $f(x)$ is increasing | 8) | intervals on which $f(x)$ is decreasing |
| 9) | relative extrema (if any) | 10) | absolute extrema (if any) |
| 11) | intervals of concavity | 12) | inflection points (if any) |