

King Fahd University of Petroleum & Minerals  
Department of Mathematical Sciences

Math 101 - 2 & 7  
Dr. Jawad Y. Abuhlail

CODE 2

Final Exam

Semester 031

Maximum Time Allowed: 3 Hours

Name:

ID #:

Section #:

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**Q1. (10 Points) (Suggested time: 10 minutes)** State if each of the following statements is TRUE or FALSE:

1. A function  $f(x)$  that is decreasing on  $(a, \infty)$  for some  $a \in \mathbb{R}$  can have no absolute minimum.
2.  $\log_b x$  is increasing for every  $b > 0$  ( $b \neq 1$ ).
3. A relative minimum of a function may be larger than a relative maximum of that function.
4. A function  $f(x)$  that is continuous on  $[a, b]$  and differentiable on  $(a, b)$  has exactly one  $c \in (a, b)$ , such that  $f'(c) = \frac{f(b)-f(a)}{b-a}$
5. Newton's Method can only converge to the right root..

**Q2. (10 Points) (Suggested time: 15 minutes)** Find the area of the largest rectangle that can be inscribed in an equilateral triangle (with equal sides), the length of each of its sides is 4 inch.

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**Q3. (60 Points) (Suggested time: 100 minutes) Encircle the most correct answer:**

1. If a particle is moving on the  $s$ -axis and its position versus time is given by  $s(t) = t^3 - 3t^2 + 4$ ,  $0 \leq t \leq 3$ . Then the particle is slowing down on

- (a)  $(0, 1) \cup (2, 3)$
- (b)  $(2, 3)$
- (c)  $(1, 2)$
- (d)  $(0, 1)$
- (e) none of the above

2. The slope of the tangent to the graph of  $x^2y^7 - x^3y^2 = 2$  at  $(-1, 1)$  is:

- (a)  $-1$
- (b)  $1$
- (c)  $\frac{-5}{9}$
- (d)  $\frac{5}{9}$
- (e) none of the above

3. The set of values of  $c$  obtained by applying the Mean Value Theorem to  $f(x) = \frac{1}{1-x}$  on  $[3, 4]$  is:

- (a)  $\{1 + \sqrt{6}, 1 - \sqrt{6}\}$
- (b)  $\{1 - \sqrt{6}\}$
- (c)  $\{1 + \sqrt{6}\}$
- (d)  $\Phi$  (i.e. there is no such  $c$ )
- (e) none of the above

4. The largest  $\delta$ , such that

$$|x - 2| < \delta \Rightarrow |x^2 - 4| < \epsilon$$

is given by:

- (a)  $\delta = \min\{1, \frac{\epsilon}{5}\}$
- (b)  $\delta = \frac{\epsilon}{5}$
- (c)  $\delta = \min\{1, \frac{\epsilon}{3}\}$
- (d)  $\delta = \frac{\epsilon}{3}$
- (e) none of the above

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5. If  $f(x) = \frac{1}{x^2-1}$  and  $g(x) = \sqrt{1+x}$ , then the domain of  $(g \circ f)(x)$  is
- (a)  $[-1, 1]$
  - (b)  $(-1, 1)$
  - (c)  $(-\infty, -1) \cup (1, \infty)$
  - (d)  $(-\infty, -1) \cup (1, \infty) \cup \{0\}$
  - (e) none of the above
6. Let  $f(x) = 2x - x^2$ ,  $x \geq 1$ . Then
- (a)  $f^{-1}(x) = 1 + \sqrt{1-x} : (-\infty, 1] \rightarrow [1, \infty)$ .
  - (b)  $f^{-1}(x) = 1 + \sqrt{1-x} : (-\infty, 0] \rightarrow [1, \infty)$ .
  - (c)  $f^{-1}(x) = 1 - \sqrt{1-x} : (-\infty, 1] \rightarrow [1, \infty)$ .
  - (d)  $f^{-1}(x) = 1 - \sqrt{1-x} : (-\infty, 0] \rightarrow [1, \infty)$ .
  - (e) none of the above
7. The graph of  $f(x) = \frac{x^4-x^3-x+1}{x^3-x}$  has
- (a) oblique asymptote  $y = x - 1$  and two vertical asymptotes:  $x = 0$ ,  $x = -1$ .
  - (b) horizontal asymptote  $y = 1$
  - (c) oblique asymptote  $y = x + 1$
  - (d) oblique asymptote  $y = x - 1$  and three vertical asymptotes  $x = 0$ ,  $x = 1$ ,  $x = -1$ .
  - (e) none of the above
8.  $\lim_{x \rightarrow -\infty} (x + \ln(x^2 + 3)) =$
- (a) 1
  - (b)  $-\infty$
  - (c)  $\infty$
  - (d) 0
  - (e) none of the above

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9. Applying Newton's Theorem to  $f(x) = x^2 - 2$  with  $x_1 = 1$ , the value of  $x_3$  will be

- (a)  $-\frac{9}{4}$
- (b)  $-\frac{17}{12}$
- (c)  $\frac{9}{4}$
- (d)  $\frac{17}{12}$
- (e) none of the above

10. The Local Linear Approximation of  $f(x) = \sin^{-1}(x)$  at  $x_0 = \frac{-1}{\sqrt{2}}$  is:

- (a)  $-\sqrt{2}x + (\frac{\pi}{4} - 1)$
- (b)  $-\sqrt{2}x + (1 - \frac{\pi}{4})$
- (c)  $\sqrt{2}x + (\frac{\pi}{4} - 1)$
- (d)  $\sqrt{2}x + (1 - \frac{\pi}{4})$
- (e) none of the above

11. The instantaneous rate of change of  $f(x) = \sec^{-1}(-(1 + x^2))$ ,  $|x| \geq 1$  is given by:

- (a)  $\frac{1}{|x|(1+x^2)\sqrt{x^2+2}}$
- (b)  $\frac{-1}{|x|(1+x^2)\sqrt{x^2+2}}$
- (c)  $\frac{2x}{\sec(-1-x^2)\tan(-1-x^2)}$
- (d)  $\frac{-2x}{\sec(-1-x^2)\tan(-1-x^2)}$
- (e) none of the above

12. The radius of a cylinder is measured with a percentage error within  $\pm 0.08\%$ , while its height is measured exactly. Then the percentage error in calculating the volume of the cylinder will then be within:

- (a)  $\pm 16\%$
- (b)  $\pm 0.16\%$
- (c)  $\pm 0.016\%$
- (d)  $\pm 0.0016\%$
- (e) none of the above

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13.  $\cos^{-1}(\cos(\frac{31\pi}{4})) =$

- (a)  $\frac{3\pi}{4}$
- (b)  $\frac{\pi}{4}$
- (c)  $\frac{-\pi}{4}$
- (d)  $\frac{31\pi}{4}$
- (e) none of the above

14.  $f(x) = x^{\frac{2}{3}}$  has

- (a) a corner at  $x_0 = 0$
- (b) a cusp at  $x_0 = 0$
- (c) a vertical tangent at  $x_0 = 0$ .
- (d) discontinuity at  $x_0 = 0$
- (e) none of the above

15.  $\lim_{x \rightarrow 0} \frac{\tan(5x)}{\sin(3x)} =$

- (a)  $\frac{-5}{3}$
- (b)  $\frac{5}{3}$
- (c)  $\frac{3}{5}$
- (d) 0
- (e) Does Not Exist

16. If the equation  $Q(t) = Q_0 e^{-kt}$  ( $k > 0$ ) gives the amount in grams of a radioactive element after  $t$  hours, then the time needed to reduce an amount of this element to half of its initial value is:

- (a)  $\frac{\ln 2}{kQ_0}$  hours
- (b)  $\frac{\ln(2Q_0)}{k}$  hours
- (c)  $\frac{k}{\ln 2}$  hours
- (d)  $\frac{\ln 2}{k}$  hours
- (e) none of the above

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17. A square is inscribed in a circle, so that each of its corners lie on the circumference of the circle. If the radius of the circle increases at a rate of  $2 \text{ inch/hour}$ , when  $r = 10 \text{ inch}$ , then the rate at which the area of the square is changing at that instant is
- (a)  $2\sqrt{2} \text{ inch}^2/\text{hour}$
  - (b)  $20\sqrt{2} \text{ inch}^2/\text{hour}$
  - (c)  $8 \text{ inch}^2/\text{hour}$
  - (d)  $80 \text{ inch}^2/\text{hour}$
  - (e) none of the above
18. The function  $f(x) = \frac{-x}{x^2+1}$
- (a) has an absolute maximum and an absolute minimum
  - (b) has an absolute minimum but no absolute maximum
  - (c) has an absolute maximum but no absolute minimum
  - (d) has neither an absolute maximum nor an absolute minimum
  - (e) is not bounded
19.  $\lim_{x \rightarrow -1^-} \frac{x^2-5x-7}{x+1} =$
- (a)  $\infty$
  - (b)  $-\infty$
  - (c)  $\frac{-11}{2}$
  - (d)  $-7$
  - (e) none of the above
20.  $\frac{d}{dx}((x^4 + 3)^{\cos x}) =$
- (a)  $(x^4 + 3)^{\cos x} \cdot \left(\frac{4x^3 \cos(x)}{x^4+3} - \sin(x) \ln(x^4 + 3)\right)$
  - (b)  $4x^3(x^4 + 3)^{\cos x} \ln(x^4 + 3)$
  - (c)  $(x^4 + 3)^{\cos x} \ln(x^4 + 3)$
  - (d)  $\cos x \cdot (x^4 + 3)^{\cos x - 1}$
  - (e) none of the above

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**Q5. (20 Points) (Suggested time: 25 minutes)** Consider the function:

$$f(x) = x^2 e^{-x}$$

1. **Find each of the following (Details should be included on the back).**

- (a) Domain( $f(x)$ ) =
- (b) Range( $f(x)$ ) =
- (c)  $x$ -intercept(s) (if any):
- (d)  $y$ -intercept:
- (e) Symmetries (if any):
- (f)  $\lim_{x \rightarrow \infty} f(x) =$
- (g)  $\lim_{x \rightarrow -\infty} f(x) =$
- (h) Asymptotes (if any):
- (i) Critical Points (if any):
- (j) Interval(s) on which  $f(x)$  is increasing (if any):
- (k) Interval(s) on which  $f(x)$  is decreasing (if any):
- (l) Relative Maximum (if any):
- (m) Relative Minimum (if any):
- (n) Absolute Maximum (if any):
- (o) Absolute Minimum (if any):
- (p) Intervals on which the graph of  $f(x)$  is concave up (if any):
- (q) Intervals on which the graph of  $f(x)$  is concave down (if any):
- (r) Inflection Points (if any):

2. **Draw the graph of  $f(x)$  (The final graph should be included on this page)**