King Fahd University of Petroleum & Minerals
Department of Mathematical Sciences

Math 101 - 2 & 7

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Second Major Exam **Semester 031**
Time: 17:15-18:45 pm, Wednesday 3.12.2003

Name: **ID #:** **Section #:**

Q1. (10 Points) (Suggested time: 10 minutes) State if each of the following statements is TRUE or FALSE:

1. $f(x) = \frac{x}{2\sec(x)-1}$ is differentiable over $(-\frac{\pi}{2}, \frac{\pi}{2})$.
2. If f and $f \cdot g$ are differentiable at $x = c$, then g is also differentiable at $x = c$.
3. $f(x) = \begin{cases} x\sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is differentiable at $x = 0$.
4. The graph of $f(x) = \sqrt[5]{x}$ has a cusp at $x = 0$.
5. The function $f(x) = |x - 1|$ is differentiable on $[0, \infty)$.

Q2. (40 Points) (Suggested time: 30 minutes) Solve each of the following questions showing all details:

1. Find the equation of the normal line to the curve of $y = \frac{x^2+1}{\sin(x)+1}$ at the point $(0, 1)$.

2. Let f and g be functions such that $f(1) = -1$, $f'(1) = 2$ and $g'(-1) = -4$. Let

$$F(x) = 2(f(x))^2 - (g \circ f)(x).$$

Find $F'(1)$.

3. Find the values of c and d, so that $f(x)$ is differentiable everywhere:

$$f(x) = \begin{cases} \sqrt{1-2x}, & x \leq 0 \\ cx + d & x > 0 \end{cases}$$

4. Use Local Linear Approximation Method to estimate $\csc(46°)$.

Q3. (**20 Points**) (**Suggested time: 10 minutes**)

1. Consider the curve C given by the equation:

$$x^2 \sin(y + \pi) + x(y + \frac{\pi}{2})^2 = 1.$$

   Find the equation of the tangent line to the curve at the point $(1, -\frac{\pi}{2})$.

2. Given that $x^2 - xy^2 = 1$. Find $\frac{d^2y}{dx^2}$ (**in simplest form**).

Q4. (10 Points) (<u>Suggested time: 10 minutes</u>) Use <u>definition</u> to discuss the differentiability of $f(x)$ at $x = 1$, where

$$f(x) = \begin{cases} x^2 - x, & x \leq 1 \\ 1 - \frac{1}{x}, & x > 1 \end{cases}$$

Q5. (10 Points) (<u>Suggested time: 10 minutes</u>) A point P is moving along the right part of the parabola $y = x^2$ away from the origin. When P arrives at $(1, 1)$, its distance form the origin is increasing at a rate of $\sqrt{8}$ units/s. How fast is y changing at that instant?

Q6. (10 Points) (<u>Suggested time: 10 minutes</u>)

1. Let $y = x^n$, where $n = \frac{p}{q}$ (p, q are integers, $q \neq 0$). Show that $\frac{dy}{dx} = nx^{n-1}$. (**Hint**: assume the result is true for integer powers).

2. Give a *counter example* to the following **false** statement:

If f and g are continuous everywhere and differentiable at $x = c$, then $g \circ f$ is differentiable at $x = c$.

GOOD LUCK