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An exact solution of a quasilinear Fisher equation in cylindrical coordinates

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Abstract

Lie symmetry method is applied to analyse Fisher equation in cylindrical coordinates. Symmetry algebra is found and symmetry invariance is used to reduce the equation to a first-order ODE. The first-order ODE is further analysed to obtain exact solution of Fisher equation in explicit form.

Keywords: Partial differential equations; Fisher equation; Exact solutions

The Lie symmetry approach to study a non-linear partial differential equation is based on finding its symmetry algebra which leads to reductions and construction of exact solutions of the PDE under investigation. Symmetry solutions of Fisher equation [6]

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = u(1-u),$$

which arises in Heat Transfer, Biology & Ecology, have been investigated in [1,3,5]. It is well known for heat equation models that, in general, the thermal properties of the medium are not constant and may depend upon the temperature [7]. This leads to the following generalization:

$$\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right) = u(1-u). \tag{1}$$

If $\underline{\nabla}$ denotes the gradient operator then, in higher dimensions, this equation takes the form

$$\frac{\partial u}{\partial t} - \underline{\nabla}\cdot\left(u\frac{\underline{\nabla} u}{\partial x}\right) = u(1-u), \tag{2}$$

where complete symmetry analysis of this equation needs to be carried out. Here, we investigate Eq. (2) in case of cylindrical domains with u depending only on radius. If the radial variable is denoted by x then the corresponding

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