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ORIGINAL ARTICLE

Exact solutions of some general nonlinear wave equations in elasticity

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Abstract A similarity analysis of a nonlinear wave equation in elasticity is studied; in particular, one with anharmonic corrections. The symmetry transformation give rise to exact solutions via the method of invariants. In some cases, graphical figure of the solutions are presented. Furthermore, we consider some cases wherein the velocities of the longitudinal and transversal plane waves are variable. Finally, a brief discussion on how a symmetry analysis on a perturbation of the elasticity equation can be pursued.

Keywords Similarity analysis · Exact solutions · Nonlinear elasticity equations

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1 Introduction

The linear theory of elasticity is based upon the assumption that the strain tensor u_{ij} depends linearly on the displacement vector u_i as

$$u_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{1.1}$$

The elastic energy E in this case is given by

$$E = \int \left(\frac{\lambda}{2}u_{ii}^2 + \mu u_{ij}^2\right) \mathrm{d}\mathbf{r} \tag{1.2}$$

where $\mathbf{r}$ is the position vector of the point x_i and λ and μ are Lame's coefficients. If the displacement in the elastic medium is not small, the nonlinear strain tensor takes the form

$$u_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}). \tag{1.3}$$

Moreover, the elastic energy up to third-order takes the form

$$E = \int \left(\frac{\lambda}{2}u_{ii}^2 + \mu u_{ij}^2 + \frac{1}{3}Au_{ij}u_{jk}u_{ki} + Bu_{ij}^2u_{kk} + \frac{1}{3}Cu_{ii}^3\right) \mathrm{d}\mathbf{r} \tag{1.4}$$