Introduction to Graphs

• What is a Graph?

• Some Example applications of Graphs.

• Graph Terminologies.

• Representation of Graphs.
  – Adjacency Matrix.
  – Adjacency Lists.
  – Simple Lists

• Review Questions.
What is a Graph?

- Graphs are Generalization of Trees.

- A simple graph $G = (V, E)$ consists of a non-empty set $V$, whose members are called the vertices of $G$, and a set $E$ of pairs of distinct vertices from $V$, called the edges of $G$. 

[Diagram showing undirected, directed (Digraph), and weighted graphs]
Some Example Applications of Graph

• Finding the least congested route between two phones, given connections between switching stations.

• Determining if there is a way to get from one page to another, just by following links.

• Finding the shortest path from one city to another.

• As a traveling sales-person, finding the cheapest path that passes through all the cities that the sales person must visit.

• Determining an ordering of courses so that prerequisite courses are always taken first.
Graphs Terminologies

- **Adjacent Vertices:** there is a connecting edge.

- **A Path:** A sequence of adjacent vertices.

- **A Cycle:** A path in which the last and first vertices are adjacent.

- **Connected graph:** There is a path from any vertex to every other vertex.
More Graph Terminologies

- Path and cycles in a digraph: must move in the direction specified by the arrow.

- Connectedness in a digraph: strong and weak.

- Strongly Connected: If connected as a digraph - following the arrows.

- Weakly connected: If the underlying undirected graph is connected (i.e. ignoring the arrows).

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**Directed Cycle**

**Strongly Connected**

**Weakly Connected**
Further Graph Terminologies

• Emanate: an edge $e = (v, w)$ is said to emanate from $v$.  
  – $A(v)$ denotes the set of all edges emanating from $v$.

• Incident: an edge $e = (v, w)$ is said to be incident to $w$.  
  – $I(w)$ denote the set of all edges incident to $w$.

• Out-degree: number of edges emanating from $v$ -- $|A(v)|$

• In-degree: number of edges incident to $w$ -- $|I(w)|$.  

![Directed Graph](image1)

![Undirected Graph](image2)
Graph Representations

• For vertices:
  – an array or a linked list can be used

• For edges:
  – Adjacency Matrix (Two-dimensional array)
  – Adjacency List (One-dimensional array of linked lists)
  – Linked List (one list only)
Adjacency Matrix Representation

- Adjacency Matrix uses a 2-D array of dimension $|V| \times |V|$ for edges. (For vertices, a 1-D array is used)

- The presence or absence of an edge, $(v, w)$ is indicated by the entry in row $v$, column $w$ of the matrix.

- For an unweighted graph, boolean values could be used.

- For a weighted graph, the actual weights are used.
Notes on Adjacency Matrix

- For undirected graph, the adjacency matrix is always symmetric.

- In a Simple Graph, all diagonal elements are zero (i.e. no edge from a vertex to itself).

- The space requirement of adjacency matrix is $O(n^2)$ - most of it wasted for a graph with few edges.

- However, entries in the matrix can be accessed directly.
Adjacency List Representation

- This involves representing the set of vertices adjacent to each vertex as a list. Thus, generating a set of lists.

- This can be implemented in different ways.
- Our representation:
  - Vertices as a one dimensional array
  - Edges as an array of linked list (the emanating edges of vertex 1 will be in the list of the first element, and so on, ...
Simple List Representation

- Vertices are represented as a 1-D array or a linked list
- Edges are represented as one linked list
  - Each edge contains the information about its two vertices
Review Questions

1. Consider the undirected graph $G_A$ shown above. List the elements of $V$ and $E$. Then, for each vertex $v$ in $V$, do the following:
   1. Compute the in-degree of $v$
   2. Compute the out-degree of $v$
   3. List the elements of $A(v)$
   4. List the elements of $I(v)$.

2. Consider the undirected graph $G_A$ shown above. Show how the graph is represented using adjacency matrix.
   Show how the graph is represented using adjacency lists.

3. Repeat Exercises 1 and 2 for the directed graph $G_B$ shown above.