A NEW SINGLE ASYMMETRIC ERROR CORRECTING CODE OF LENGTH 19

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ABSTRACT

A new code capable of correcting a single asymmetric error is proposed. This code is of length 19 and has 28142 codewords, improving the best known code by 110 code words.

1. INTRODUCTION

In this work, we present a new code capable of correcting a single asymmetric error. The proposed code is of asymmetric distance 2 and of length 19. The asymmetric distance of two binary vectors, x and y, of the same length is defined by

$$d_a(x,y) = max\{N(x,y), N(y,x)\}$$

where $N(x,y) = |\{i : x_i = 1 \text{and} y_i = 0\}|$, i.e. the number of positions where x has a 1 and y has a 0. The minimum asymmetric distance of a code C is defined as follows

$$D_a(C) = min\{d_a(x,y) : x, y \in C \text{and} x \neq y\}$$

A code C can correct d asymmetric errors or fewer if $D_a(C) > d$. The proposed code has asymmetric distance two so it is capable of correcting a single asymmetric error.

2. CONSTRUCTION METHOD

The construction of the proposed code (C) is based on the Cartesian product of two sets of partitioned codes, say $\{A_1, A_2, \ldots\}$ and $\{B_1, B_2, \ldots\}$, where

$$C = A_1 \times B_1 \cup A_2 \times B_2 \cup A_3 \times B_3 \cup \cdots \qquad (1)$$

These two sets are defined as follows:

Let A be the set of all the 2^p binary vectors of length p and let $A_1, A_2, \ldots, A_{p\prime}$ be a partition of A, i.e. $A_i \bigcap A_j = \phi$ and $\bigcup A_i = A$, such that $D_a(A_i) \geq 2$ for $1 \leq i \leq p\prime$.

Also, let B be the set of the 2^{q-1} even weight binary vectors of length q and $B_1, B_2, \ldots, B_{q\prime}$ be a partition of B such that $D_a(B_j) \geq 2$ for $1 \leq j \leq q\prime$.

It was shown in [1] that the code constructed using (1) above, is of asymmetric distance two. The length of the constructed code is $p+q$ and the cardinality of the code is:

$$|C| = |A_1| * |B_1| + |A_2| * |B_2| + |A_3| * |B_3| + \cdots$$

Example:

To construct a single asymmetric error correcting code with $n = 6$, let $p = 2$ and $q = 4$. Then $A = \{00, 01, 10, 11\}$ can be partitioned into $A_1 = \{00, 11\}, A_2 = \{01\}$, and $A_3 = \{10\}$. And, $B = \{0000, 0011, 0101, \ldots, 1111\}$ can be partitioned into $B_1 = \{0000, 0011, 1100, 1111\}$, $B_2 = \{0101, 1010\}$, and $B_3 = \{0110, 1001\}$.

We obtain a code C of length 6 where $C = A_1 \times B_1 \cup A_2 \times B_2 \cup A_3 \times B_3$ having $2*4+1*2+1*2 = 12$ code words as shown in Fig.1.

00	0000	
00	0011	
00	1100	
00	1111	$A_1 \times B_1$
11	0000	
11	0011	
11	1100	
11	1111	
01	0101	
01	1010	$A_2 \times B_2$
10	0110	
10	1001	$A_3 \times B_3$

Fig. 1: Constructing a single asymmetric error correcting code for $n = 6$.

3. THE NEW CODE OF LENGTH 19

The best known single asymmetric error correcting code of length 19 and 28032 code words was given in [1]. It was obtained from the Cartesian product of two partitions, A and B. The A partition contains all binary vectors of length $p = 7$ and it is obtained from the Abelian group Z_8 (see [4]); yielding 8 partitions each of size 16 code words, i.e. $A = \{A_1, A_2, \ldots A_8\}$ where $|A_i| = 16$ for all i. The B partition conatins all the even binary vectors of length $q = 12$ and it has 11 partitions $B = \{B_1, B_2, \ldots B_{11}\}$ of the following eleven sizes: 248, 246, 234, 234, 224, 198, 192, 176, 136, 94, and 66, respectively (see [3]).

Using the Cartesian product method with these A and B partitions, the code of length 19 and of size: 16 * (248 + 246 + 234 + 234 + 224 + 198 + 192 + 176) + 0 * (136 + 94 + 66) = 28032 code words, was constructed in [1].

Here, we obtain a new A partition of length 7 with 9 different partitions of the following sizes: 18, 18, 16, 16, 16, 15, 14, 12, and 3. These partitions are shown in Fig.2. Applying the Cartesian product method using the new A partition with the above B partition gives a new code of length 19 and of size: $18*(248+246)+16*(234+234+224)+15*198+14*192+12*176+3*136+0*(94+66) = 28142$ code words. This code improves the best known code by 110 code words.

A_1	A_2	A_3	A_4
0000001	0000100	0010000	0000000
0001010	0010001	0100001	0010100
0100100	0101000	1000100	1100000
1010000	1000010	0000111	0001011
0010101	0001101	0011001	0100101
0100011	0011010	0110100	0110010
0111000	0100110	1010010	1001100
1000110	1010100	1101000	1010001
1001001	1100001	0011110	0011101
0001111	0110011	0101011	0101110
0110110	0111100	1001101	1010110
1011010	1000111	1100110	1100011
1101100	1011001	0110111	1111000
1110001	1101010	1011011	1001111
0111011	0011111	1111100	1110101
1011101	1101101	1101111	1111111
1100111	1110110		
1111110	1111011		

A_5	A_6	A_7	A_8
0000010	0001000	0100000	1000000
0000101	0010010	0000011	0001100
0110000	1000001	0011000	0100010
1001000	0001110	0010110	0010011
0011100	0110001	0101001	0111001
0101010	1011000	1000101	1001110
1000011	1100010	1001010	1100101
1100100	0010111	1110000	1110010
0011011	0101101	0110101	0111110
0100111	0111010	1010011	1010111
1010101	1001011	1011100	1101011
1101001	1110100	0101111	1111101
0111101	1011110	1111010	
1101110	1111001	1110111	
1110011	0111111		
1011111			

A_9
0000110
0001001
0101100

Fig. 2: The new A partition of the 2^7 binary vectors.

The new A partition is obtained by graph-coloring method. In this method, we construct a graph $G = (N, E)$, where N is the set of nodes and E is the set of edges in the graph, as follows:

N is the set of all 2^7 binary vectors and

$E = \{(x, y) : x, y \in N; D_a(x, y) = 1\}$

The nodes of the graph are colored using 9 colors, i.e. $\forall x, y \in N$, if $(x, y) \in E$ then x and y have different colors. Cearly, the set of all nodes having color k, say A_k, satisfy $D_a(A_k) \geq 2$.

4. ACKNOWLEDGEMENTS

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5. REFERENCES

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