Chapter 5: Valuing Bonds

9. Bond 1

$$\text{Year 1: PV} = \$80 \times \left[\frac{1}{0.10} - \frac{1}{0.10(1.10)^{10}}\right] + \frac{\$1{,}000}{1.10^{10}} = \$877.11$$

$$\text{Year 2: PV} = \$80 \times \left[\frac{1}{0.10} - \frac{1}{0.10(1.10)^{9}}\right] + \frac{\$1{,}000}{1.10^{9}} = \$884.82$$

Using a financial calculator:

Year 1: PMT = 80, FV = 1000, i = 10%, n = 10; compute PV_0 = $877.11

Year 2: PMT = 80, FV = 1000, i = 10%, n = 9; compute PV_1 = $884.82

$$\text{Rate of return} = \frac{\$80 + (\$884.82 - \$877.11)}{\$877.11} = 0.100 = 10.0\%$$

Bond 2

$$\text{Year 1: PV} = \$120 \times \left[\frac{1}{0.10} - \frac{1}{0.10(1.10)^{10}}\right] + \frac{\$1{,}000}{1.10^{10}} = \$1{,}122.89$$

$$\text{Year 2: PV} = \$120 \times \left[\frac{1}{0.10} - \frac{1}{0.10(1.10)^{9}}\right] + \frac{\$1{,}000}{1.10^{9}} = \$1{,}115.18$$

Using a financial calculator:

Year 1: PMT = 120, FV = 1000, i = 10%, n = 10; compute PV_0 = $1,122.89

Year 2: PMT = 120, FV = 1000, i = 10%, n = 9; compute PV_1 = $1,115.18

$$\text{Rate of Return} = \frac{\$120 + (\$1{,}115.18 - \$1{,}122.89)}{\$1{,}122.89} = 0.100 = 10.0\%$$

Both bonds provide the same rate of return.

10. a. If yield to maturity = 8%, price will be $1,000.

b. Rate of return =

$$\frac{\text{coupon income} + \text{price change}}{\text{investment}} = \frac{\$80 + (\$1{,}000 - \$1{,}100)}{\$1{,}100} = -0.0182 = -1.82\%$$

c. $$\text{Real return} = \frac{1 + \text{nominal interest rate}}{1 + \text{ inflation rate}} - 1 = \frac{0.9818}{1.03} - 1 = -0.0468 = -4.68\%$$

12. a. To compute the yield to maturity, use trial and error to solve for r in the following equation:

$$900 = 80 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{30}}\right] + \frac{1{,}000}{(1+r)^{30}} \Rightarrow r = 8.971\%$$

Using a financial calculator, compute the yield to maturity by entering:
n = 30; PV = (–)900; FV = 1000; PMT = 80, compute i = 8.971%

Verify the solution as follows:

$$PV = 80 \times \left[\frac{1}{0.08971} - \frac{1}{0.08971(1.08971)^{30}}\right] + \frac{1{,}000}{1.08971^{30}} = 899.99$$

(difference due to rounding)

b. Since the bond is selling for face value, the yield to maturity = 8.000%

13. a. To compute the yield to maturity, use trial and error to solve for r in the following equation:

$$900 = 40 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{60}}\right] + \frac{1{,}000}{(1+r)^{60}} \Rightarrow r = 4.483\%$$

Using a financial calculator, compute the yield to maturity by entering:
n = 60; PV = (–)900; FV = 1000; PMT = 40, compute i = 4.483%

Verify the solution as follows:

$$PV = 40 \times \left[\frac{1}{0.04483} - \frac{1}{0.04483(1.04483)^{60}} \right] + \frac{1{,}000}{1.04483^{60}} = 900.02$$

Therefore, the annualized bond equivalent yield to maturity is:

4.483% × 2 = 8.966%

b. Since the bond is selling for face value, the semi-annual yield = 4%

Therefore, the annualized bond equivalent yield to maturity is: 4% × 2 = 8%

14. In each case, we solve the following equation for the missing variable:

Price = \$1,000/(1 + y)$^{\text{maturity}}$

Price	Maturity (Years)	Yield to Maturity
\$300.00	30.00	4.095%
\$300.00	15.64	8.000%
\$385.54	10.00	10.000%

15. PV of perpetuity = coupon payment/rate of return.

PV = C/r = \$60/0.06 = \$1,000.00

If the required rate of return is 10%, the bond sells for:

PV = C/r = \$60/0.10 = \$600.00

16. Current yield = 0.098375 so bond price can be solved from the following:

\$90/Price = 0.098375 ⇒ Price = \$914.87

To compute the remaining maturity, solve for t in the following equation:

$$\$914.87 = \$90 \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^{t}} \right] + \frac{\$1{,}000}{(1.10)^{t}} \Rightarrow t = 20.0$$

Using a financial calculator, compute the remaining maturity by entering:
PV = (–)914.87; FV = 1000; PMT = 90, i = 10 and compute n = 20.0 years.

17. a. The coupon rate must be 7% because the bonds were issued at face value with a yield to maturity of 7%. Now, the price is:

$$PV = \$70 \times \left[\frac{1}{0.15} - \frac{1}{0.15(1.15)^8}\right] + \frac{\$1{,}000}{1.15^8} = \$641.01$$

b. The investors pay $641.01 for the bond. They expect to receive the promised coupons plus $800 at maturity. We calculate the yield to maturity based on these expectations by solving the following equation for r:

$$\$641.01 = \$70 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^8}\right] + \frac{\$800}{(1+r)^8} \Rightarrow r = 12.87\%$$

Using a financial calculator, enter: n = 8; PV = (–)641.01; FV = 800; PMT = 70, and then compute i = 12.87%

20. a. At a price of $1,100 and remaining maturity of 9 years, find the bond's yield to maturity by solving for r in the following equation:

$$\$1{,}100 = \$80 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^9}\right] + \frac{\$1{,}000}{(1+r)^9} \Rightarrow r = 6.50\%$$

Using a financial calculator, enter: n = 9; PV = (–)1100; FV = 1000; PMT = 80, and then compute i = 6.50%

b. Rate of return $= \frac{\$80 + (\$1{,}100 - \$980)}{\$980} = 20.41\%$

21. a, b.

Price of each bond at different yields to maturity

	Maturity of bond		
Yield	4 years	8 years	30 years
7%	$1,033.87	$1,059.71	$1,124.09
8%	$1,000.00	$1,000.00	$1,000.00
9%	$967.60	$944.65	$897.26

c. The table shows that prices of longer-term bonds are more sensitive to changes in interest rates.

23. The bond's yield to maturity will increase from 7.5% to 7.8% when the perceived default risk increases. The bond price will fall:

$$\text{Initial Price} = \text{PV} = \$70 \times \left[\frac{1}{0.075} - \frac{1}{0.075(1.075)^{10}}\right] + \frac{\$1{,}000}{1.075^{10}} = \$965.68$$

$$\text{New Price} = \text{PV} = \$70 \times \left[\frac{1}{0.078} - \frac{1}{0.078(1.078)^{10}}\right] + \frac{\$1{,}000}{1.078^{10}} = \$945.83$$

25. The nominal rate of return is 6%.

The real rate of return is: [1.06/(1 + inflation)] – 1

a. 1.06/1.04 – 1 = 0.0192 = 1.92%

b. 1.06/1.06 – 1 = 0.00 = 0%

c. 1.06/1.08 – 1 = –0.0185 = –1.85%