Chapter 4: Time Value of Money

1. a. $€10{,}000/(1.10)^{10} = €3{,}855.43$

   b. $€10{,}000/(1.10)^{20} = €1{,}486.44$

   c. $€10{,}000/(1.05)^{10} = €6{,}139.13$

   d. $€10{,}000/(1.05)^{20} = €3{,}768.89$

2. a. $\$100 \times (1.10)^{10} = \$259.37$

   b. $\$100 \times (1.10)^{20} = \$672.75$

   c. $\$100 \times (1.05)^{10} = \$162.89$

   d. $\$100 \times (1.05)^{20} = \$265.33$

5.

	Present Value	Years	Future Value	Interest Rate
a.	\$400	11	\$684	$\left[\frac{684}{400}\right]^{(1/11)} - 1 = 5.00\%$
b.	\$183	4	\$249	$\left[\frac{249}{183}\right]^{(1/4)} - 1 = 8.00\%$
c.	\$300	7	\$300	$\left[\frac{300}{300}\right]^{(1/7)} - 1 = 0\%$

To find the interest rate, we rearrange the basic future value equation as follows:

$$FV = PV \times (1 + r)^t \Rightarrow r = \left[\frac{FV}{PV}\right]^{(1/t)} - 1$$

7. PV = (2,000 pesos/1.06) + (4,000 pesos/1.06^2) + (5,000 pesos/1.06^3)

= 1,886.79 pesos + 3,559.99 pesos + 4,198.10 pesos = 9,644.88 pesos

8. You should compare the present values of the two annuities; select the annuity with the greater present value.

a. $$PV = \$1{,}000 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{10}}\right] = \$7{,}721.73$$

$$PV = \$800 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{15}}\right] = \$8{,}303.73$$

b. $$PV = \$1{,}000 \times \left[\frac{1}{0.20} - \frac{1}{0.20 \times (1.20)^{10}}\right] = \$4{,}192.47$$

$$PV = \$800 \times \left[\frac{1}{0.20} - \frac{1}{0.20 \times (1.20)^{15}}\right] = \$3{,}740.38$$

When the interest rate is low, as in part (a), the longer (i.e., 15-year) but smaller annuity is more valuable because the impact of discounting on the present value of future payments is less significant.

9. $\$100 \times (1 + r)^3 = \$115.76 \Rightarrow r = 5.00\%$

$\$200 \times (1 + r)^4 = \$262.16 \Rightarrow r = 7.00\%$

$\$100 \times (1 + r)^5 = \$110.41 \Rightarrow r = 2.00\%$

10. In these problems, you can either solve the equation provided directly, or you can use your financial calculator, setting: PV = (–)400, FV = 1000, PMT = 0, i as specified by the problem. Then compute n on the calculator.

a. $\$400 \times (1.04)^t = \$1{,}000 \Rightarrow t = 23.36$ periods

b. $\$400 \times (1.08)^t = \$1{,}000 \Rightarrow t = 11.91$ periods

c. $\$400 \times (1.16)^t = \$1{,}000 \Rightarrow t = 6.17$ periods

11.

	APR	Compounding period	Effective annual rate
a.	12%	1 month (m = 12/yr)	$1.01^{12} - 1 = 0.1268 = 12.68\%$
b.	8%	3 months (m = 4/yr)	$1.02^{4} - 1 = 0.0824 = 8.24\%$
c.	10%	6 months (m = 2/yr)	$1.05^{2} - 1 = 0.1025 = 10.25\%$

12.

	Effective Rate	Compounding period	Per period rate	APR
a.	10.00%	1 month (m = 12/yr)	$1.10^{(1/12)} - 1 = 0.0080$	0.096 = 9.6%
b.	6.09%	6 months (m = 2/yr)	$1.0609^{(1/2)} - 1 = 0.0300$	0.060 = 6.0%
c.	8.24%	3 months (m = 4/yr)	$1.0824^{(1/4)} - 1 = 0.0200$	0.080 = 8.0%

14. APR = 1% × 52 = 52%

EAR = $(1.01)^{52} - 1 = 0.6777 = 67.77\%$

15. Semiannual compounding means that the 8.6 percent loan really carries interest of 4.3 percent per half year. Similarly, the 8.4 percent loan has a *monthly* rate of 0.7 percent.

APR	Compounding period	Effective annual rate
8.6%	6 months (m = 2/yr)	$1.043^{2} - 1 = 0.0879 = 8.79\%$
8.4%	1 month (m = 12/yr)	$1.007^{12} - 1 = 0.0873 = 8.73\%$

Choose the 8.4 percent loan for its slightly lower effective rate.

19. The present value of the first offer is:

¥5,000,000 + ¥5,000,000/1.05 = ¥9,761,904.76

The first offer has the higher present value.

22. If the payment is denoted C, then:

$$C \times \left[\frac{1}{(0.10/12)} - \frac{1}{(0.10/12) \times [1 + (0.10/12)]^{48}} \right] = \$8{,}000 \Rightarrow C = PMT = \$202.90$$

The monthly interest rate is: 0.10/12 = 0.008333 = 0.8333 percent

Therefore, the effective annual interest rate on the loan is:

$(1.008333)^{12} - 1 = 0.1047 = 10.47$ percent

26. a. With PV = \$9,000 and FV = \$10,000, the annual interest rate is determined by solving the following equation for r:

$\$9{,}000 \times (1 + r) = \$10{,}000 \Rightarrow r = 11.11\%$

b. The present value is: $\$10{,}000 \times (1 - d)$

The future value to be paid back is \$10,000.

Therefore, the annual interest rate is determined as follows:

$PV \times (1 + r) = FV$

$[\$10{,}000 \times (1 - d)] \times (1 + r) = \$10{,}000$

$$1 + r = \frac{1}{1-d} \Rightarrow r = \frac{1}{1-d} - 1 = \frac{d}{1-d} > d$$

c. The discount is calculated as a fraction of the future value of the loan. In fact, the proper way to compute the interest rate is as a fraction of the funds borrowed. Since PV is less than FV, the interest payment is a smaller fraction of the future value of the loan than it is of the present value. Thus, the true interest rate exceeds the stated discount factor of the loan.

27. a. If we assume cash flows come at the end of each period (ordinary annuity) when in fact they actually come at the beginning (annuity due), we discount each cash flow by one period too many. Therefore we can obtain the PV of an annuity due by multiplying the PV of an ordinary annuity by (1 + r).

b. Similarly, the FV of an annuity due equals the FV of an ordinary annuity times (1 + r). Because each cash flow comes at the beginning of the period, it has an extra period to earn interest compared to an ordinary annuity.

28. Use trial-and-error to solve the following equation for r:

$$\$240 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{48}}\right] = \$8{,}000 \Rightarrow r = 1.599\%$$

Using a financial calculator, enter: PV = (–)8000; n = 48; PMT = 240; FV = 0, then compute r = 1.599% per month.

APR = 1.599 % × 12 = 19.188%

The effective annual rate is: $(1.01599)^{12} - 1 = 0.2097 = 20.97\%$

29. The annual payment over a four-year period that has a present value of $8,000 is computed by solving the following equation for C:

$$C \times \left[\frac{1}{0.2097} - \frac{1}{0.2097 \times (1.2097)^{4}}\right] = \$8{,}000 \Rightarrow C = PMT = \$3{,}147.29$$

[Using a financial calculator, enter: PV = (–)8000, n = 4, FV = 0, i = 20.97, and compute PMT.] With monthly payments, you would pay only $240 × 12 = $2,880 per year. This value is lower because the monthly payments come before year-end, and therefore have a higher PV.

31. Compare the present value of the payments. Assume the product sells for $100.

Installment plan:

PV = $25 + [$25 × annuity factor(5%, 3 years)]

$$PV = \$25 + \$25 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{3}}\right] = \$93.08$$

Pay in full: Payment net of discount = $90

Choose the second payment plan for its lower present value of payments.

32. Installment plan:

$$PV = \$25 \times \text{annuity factor}(5\%, 4 \text{ years})$$

$$PV = \$25 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^4}\right] = \$88.65$$

Now the installment plan offers the lower present value of payments.

47. The present value of your payments to the bank equals:

$$PV = \$100 \times \left[\frac{1}{0.06} - \frac{1}{0.06 \times (1.06)^{10}}\right] = \$736.01$$

The present value of your receipts is the value of a \$100 perpetuity deferred for 10 years:

$$\frac{100}{0.06} \times \frac{1}{(1.06)^{10}} = \$930.66$$

This is a good deal if you can earn 6% on your other investments.

50. a. The present value of the ultimate sales price is: \$4 million/$(1.08)^5$ = \$2.722 million

b. The present value of the sales price is less than the cost of the property, so this would not be an attractive opportunity.

c. The present value of the total cash flows from the property is now:

$$PV = [\$0.2 \text{ million} \times \text{annuity factor}(8\%, 5 \text{ years})] + \$4 \text{ million}/(1.08)^5$$

$$= \$0.2 \text{ million} \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^5}\right] + \frac{\$4 \text{ million}}{(1.08)^5} =$$

= \$0.799 million + \$2.722 million = \$3.521 million

Therefore, the property is an attractive investment if you can buy it for \$3 million.

55.

You borrow $1,000 and repay the loan by making 12 monthly payments of $100. Solve for r in the following equation:

$$\$100 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{12}}\right] = \$1{,}000 \Rightarrow r = 2.923\% \text{ per month}$$

[Using a financial calculator, enter: PV = (–)1,000, FV = 0, n = 12, PMT = 100, and compute r = 2.923%]

Therefore, the APR is: 2.923% × 12 = 35.076%

The effective annual rate is: $(1.02923)^{12} - 1 = 0.41302 = 41.302\%$

If you borrowed $1,000 today and paid back $1,200 one year from today, the true rate would be 20%. You should have known that the true rate must be greater than 20% because the twelve $100 payments are made before the end of the year, thus increasing the true rate above 20%.

60. The future value of the payments into your savings fund must accumulate to $500,000. We choose the payment (C) so that:

C × future value of an annuity = $500,000

$$C \times \left[\frac{1.06^{40} - 1}{0.06}\right] = \$500{,}000 \Rightarrow C = PMT = \$3{,}230.77$$

Using a financial calculator, enter: n = 40; i = 6; PV = 0; FV = 500,000, compute PMT = $3,230.77

62. By the time you retire you will need:

$$PV = \$40{,}000 \times \left[\frac{1}{0.06} - \frac{1}{0.06 \times (1.06)^{20}}\right] = \$458{,}796.85$$

The future value of the payments into your savings fund must accumulate to: $458,796.85 We choose the payment (C) so that:

C × future value of an annuity = $458,796.85

$$C \times \left[\frac{1.06^{40} - 1}{0.06}\right] = \$458{,}796.85 \Rightarrow C = PMT = \$2{,}964.53$$

Using a financial calculator, enter: n = 40; i = 6; PV = 0; FV = 458,796.85 and compute PMT = $2,964.53

65. (1 + nominal interest rate) = (1 + real interest rate) × (1 + inflation rate)

a. 1.03 × 1.0 = 1.03 ⇒ nominal interest rate = 3.00%

b. 1.03 × 1.04 = 1.0712 ⇒ nominal interest rate = 7.12%

c. 1.03× 1.06 = 1.0918 ⇒ nominal interest rate = 9.18%

68. a. PV = $\$100/(1.08)^3$ = \$79.38

b. real value = $\$100/(1.03)^3$ = \$91.51

c. real interest rate = $\frac{1+\text{nominal interest rate}}{1+\text{inflation rate}} - 1$ = 0.04854 = 4.854%

d. PV = $\$91.51/(1.04854)^3$ = \$79.38

77. FV = PV × $(1 + r_0) \times (1 + r_1)$ = \$1 × 1.08 × 1.10 = \$1.188

$$PV = \frac{FV}{(1+r_0)\times(1+r_1)} = \frac{\$1}{1.08\times1.10} = \$0.8418$$