Frequency Distribution Models

1- Probability Density Function (PDF)

- What is a PDF model?

    A mathematical equation that describes the frequency curve or probability distribution of a data set.

- Why modeling?
    - It represents and summarizes the statistical distribution of the entire "geologic phenomenon" and helps in making predictions.
    - Statistical characteristics and parameters can be derived and calculated easily from the model. Such parameters reflect the behavior of the "geologic phenomenon".

Frequency Distribution Models
1- Probability Density Function (PDF)

- Types of models
  - Normal distribution
    - § Gaussian (Gauss), Laplace, or Bell-shaped distribution.
    - § Values are symmetrically distributed around a central value.
    - § The mean is the most representative value of the distribution (i.e. use arithmetic average to estimate the unknown from a set of "uncorrelated random variables").
    - § The variance is the well-defined measure of the spread of observations.

Frequency Distribution Models

1- Probability Density Function (PDF)

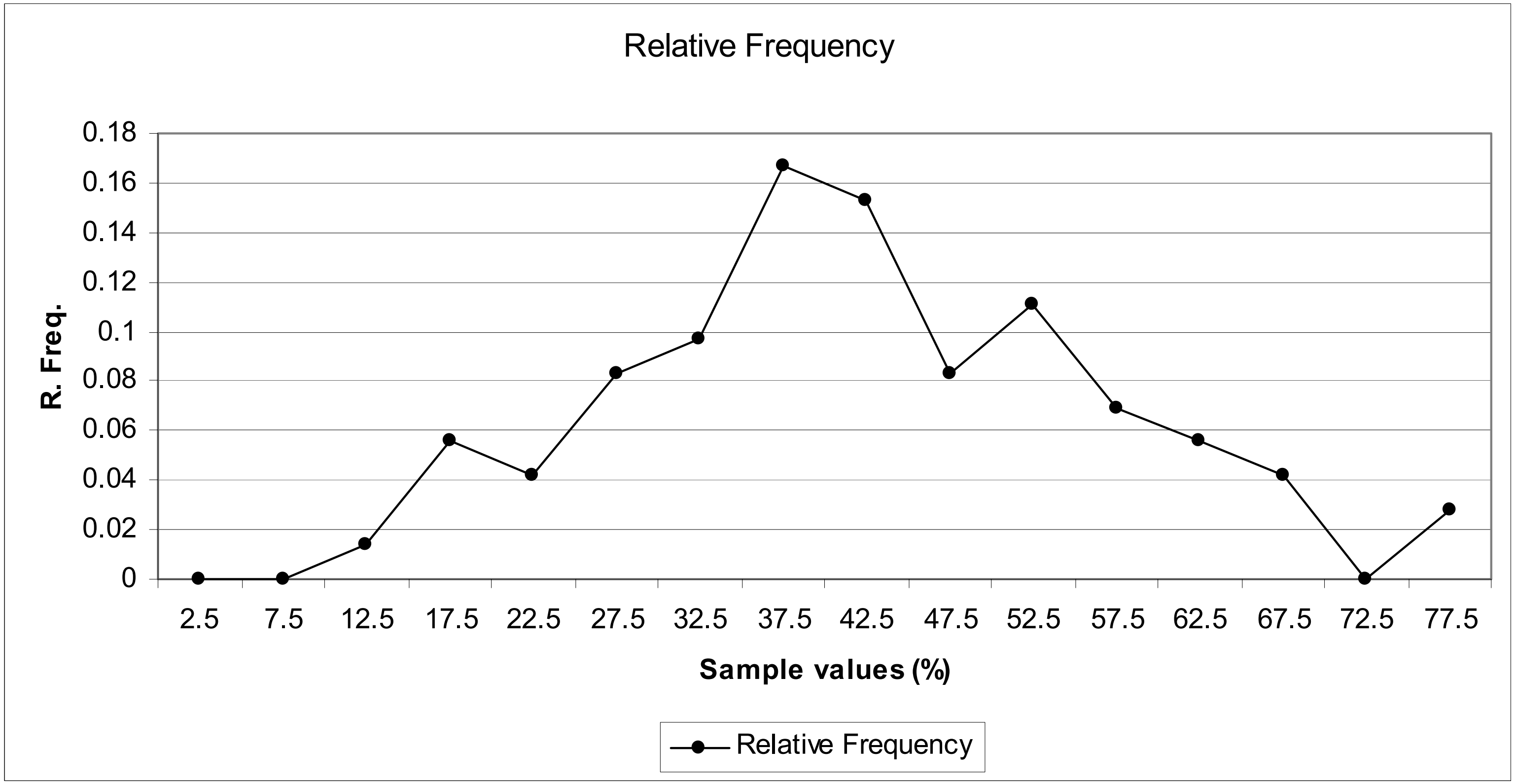
- Types of models

  – Non-normal distribution

    - Lognormal distributions (natural or base-10 logarithms of measured observations).
    - Values are asymmetrically distributed around a central value.
    - The mean is not the most representative value of the distribution. (i.e. do not simply use arithmetic average to estimate the unknown from a set of "uncorrelated random variables"). However, other averages or median value might work!

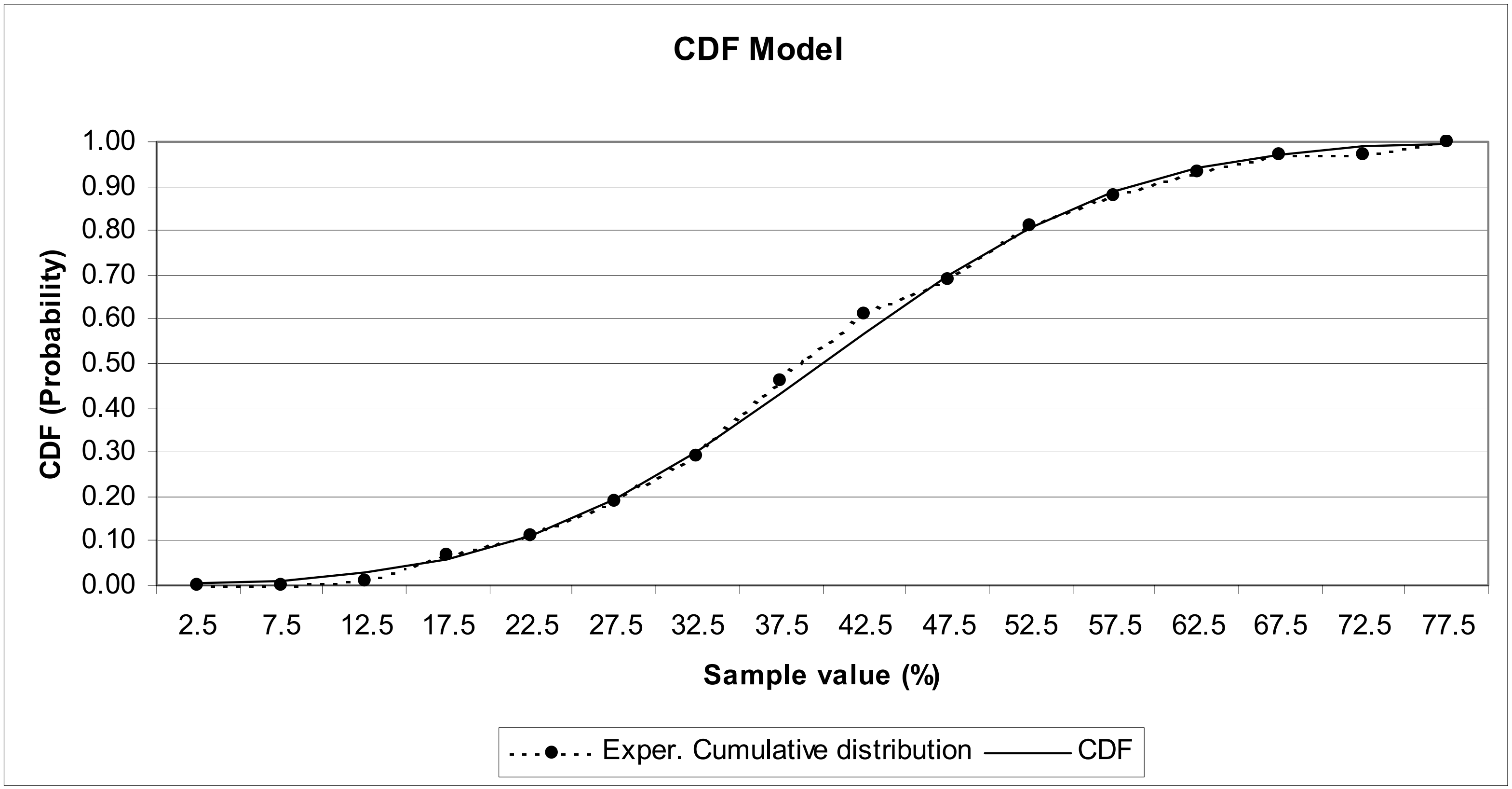
Frequency Distribution Models

2 - Probability Density Function (PDF): Example of a normal case



Frequency Distribution Models

3 - Cumulative Distribution Function (CDF): Example Example of a normal case



Frequency Distribution Models

4- Normality Tests

- Different procedures are used to test for normality. Some of these are listed below:

  » Normal-Probability graph papers

  » Chi-square test

  » Kolmogorov-Smirnov test

  » Shapiro-Wilks test

Frequency Distribution Models

5- Skewed Distributions

- What if a distribution is skewed?

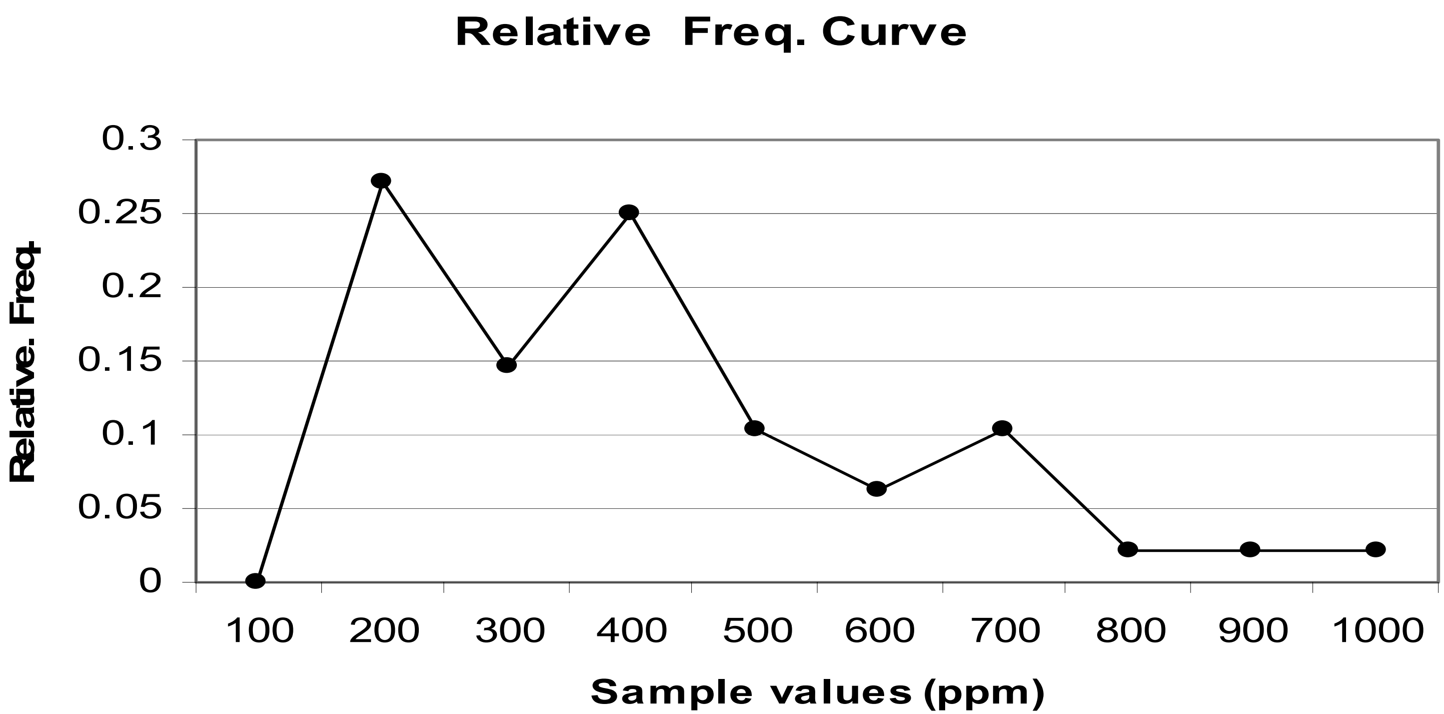
  Use power transformation techniques to transform original data values into a defined power function.

- Why and how?
  - Distribution of transformed data is much easier to describe than the distribution of original skewed data.
  - Common power transforms:

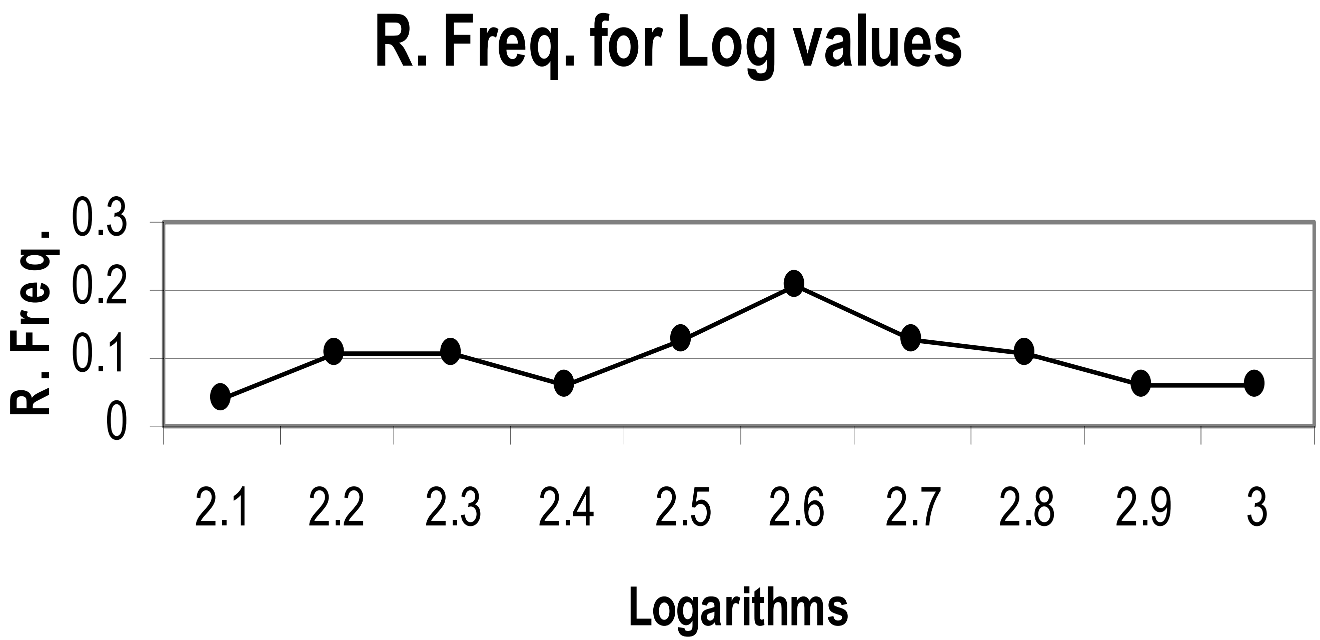
$$y = (z^p - 1) / p \qquad\qquad y = Ln(z)$$

Frequency Distribution Models

6 - PDF of original data

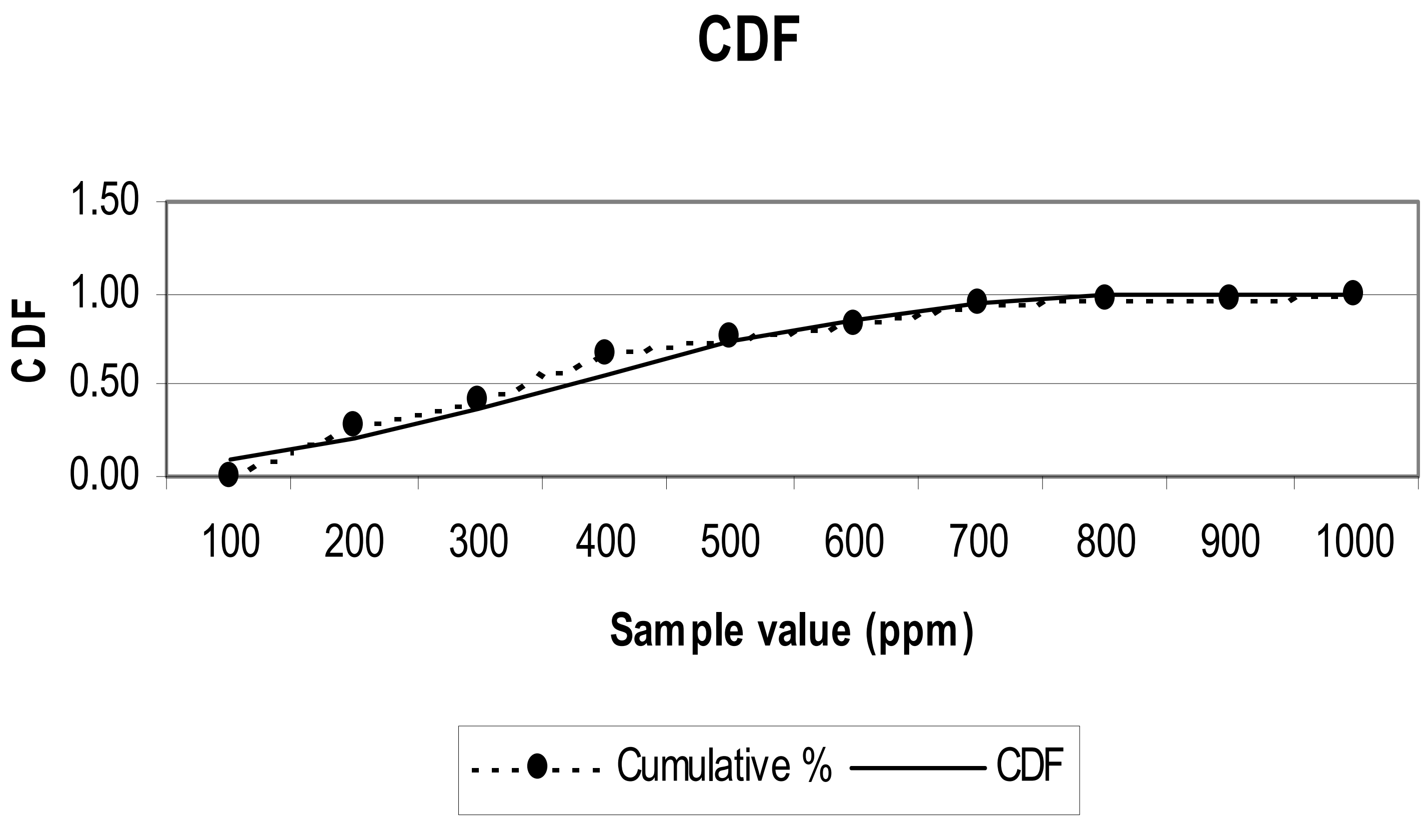


Frequency Distribution Models
6 - PDF of log-transformed data



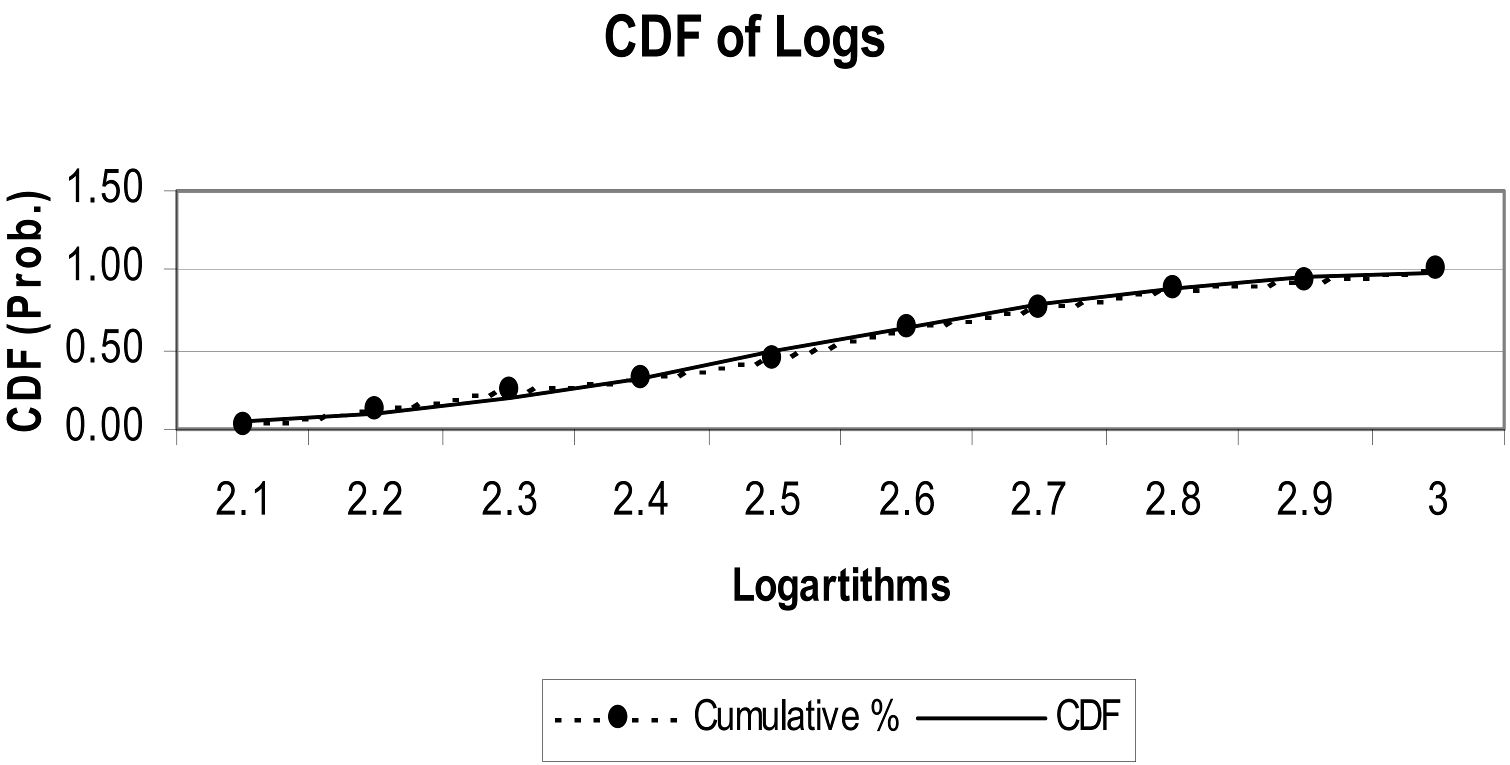
Frequency Distribution Models

6 - CDF of original data



Frequency Distribution Models

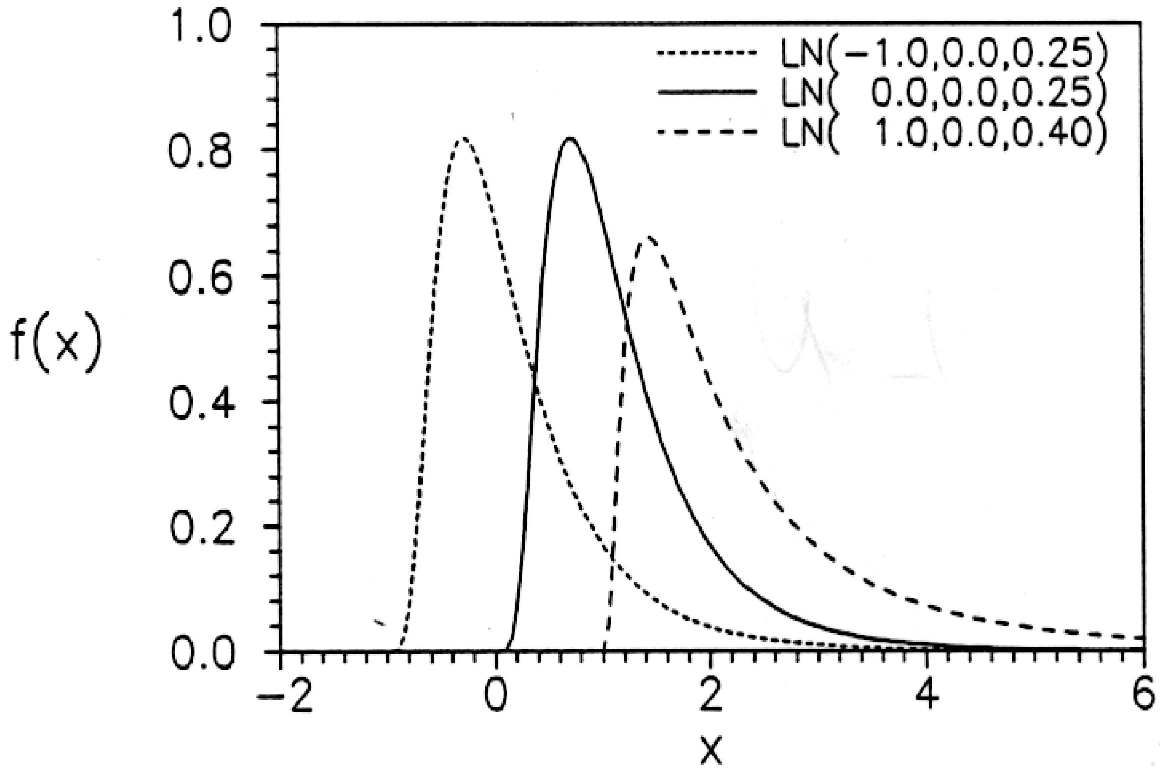
6 - CDF of log-transformed data



Frequency Distribution Models

7 - Other Types of Skewed distributions

Lognormal Model



Frequency Distribution Models

7 - Other Types of Skewed distributions

Exponential Model

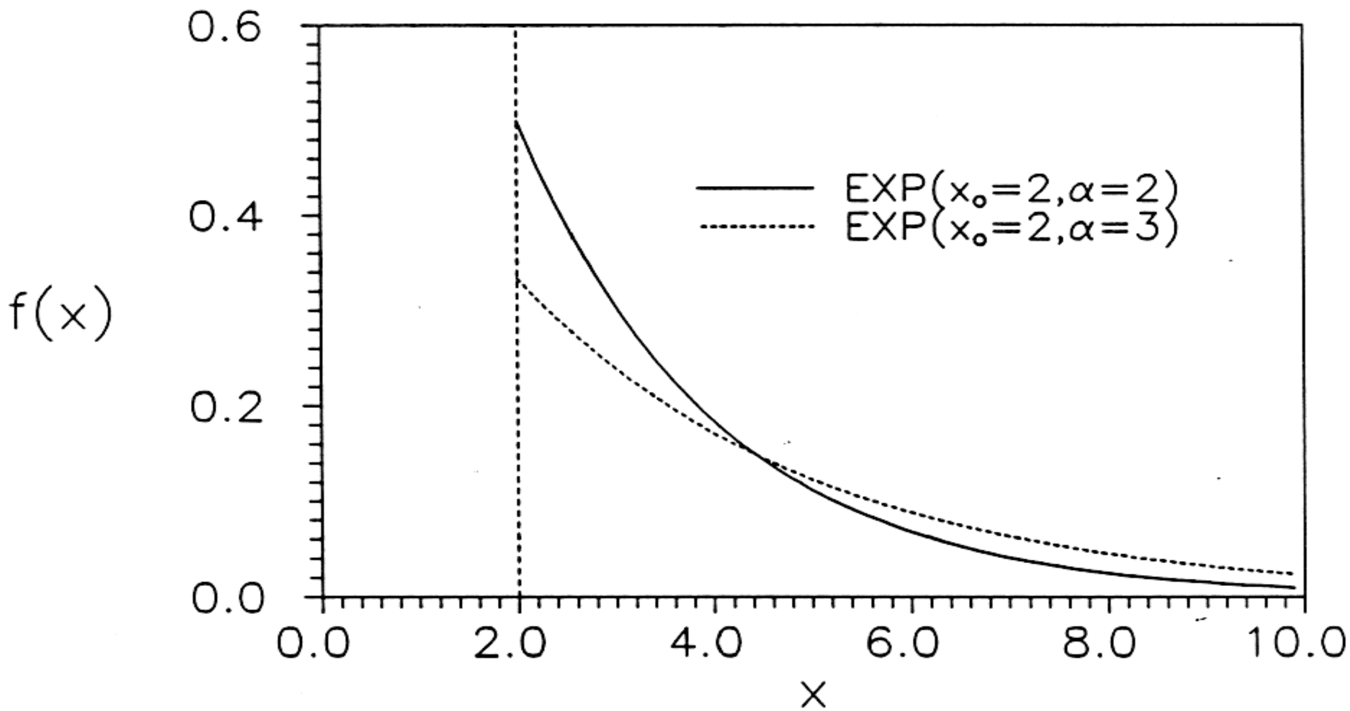
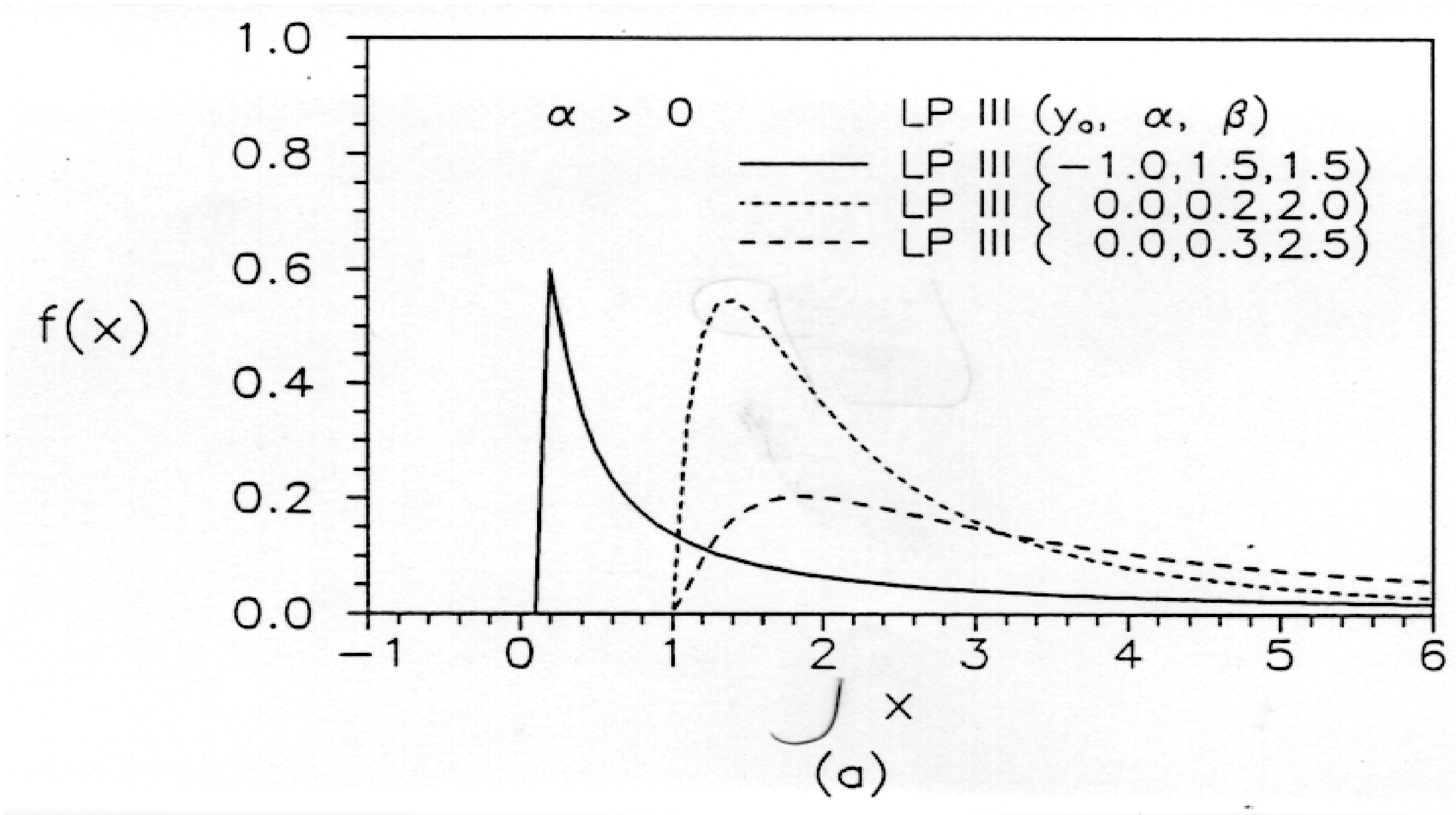


Fig. 4.7 Examples of exponential density functions with parameters (a) $x_o = 2$, $\alpha = 2$ and (b) $x_o = 2$ and $\alpha = 3$.

Frequency Distribution Models

7 - Other Types of Skewed distributions

Log-Pearson Type III Model



Frequency Distribution Models

7 - Other Types of Skewed distributions

General Extreme Value (GEV) Model

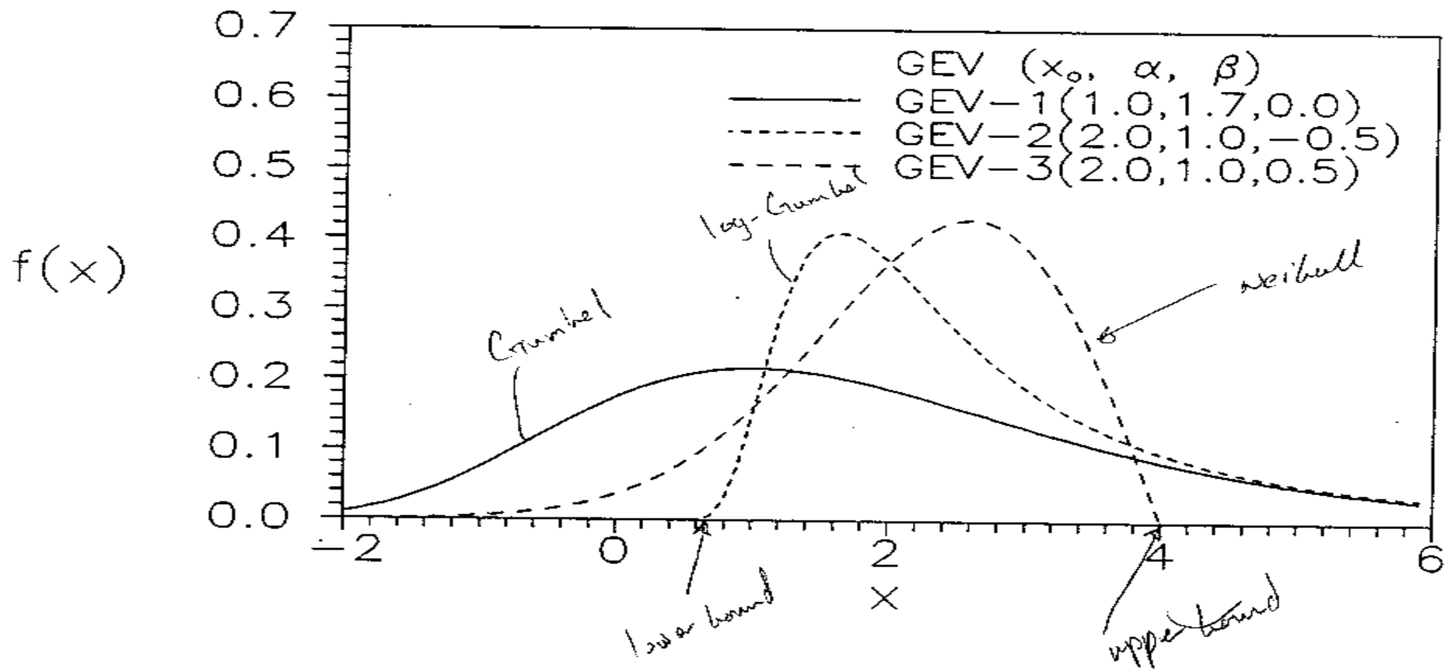


Fig. 4.14 GEV probability density functions of Eq. (4.188) with parameters (a) $x_o = 1$, $\alpha = 1.7$ and $\beta = 0$ (GEV-1); (b) $x_o = 2$, $\alpha = 1$ and $\beta = -0.5$ (GEV-2); and (c) $x_o = 2$, $\alpha = 1$ and $\beta = 0.5$ (GEV-3).

Frequency Distribution Models

7 - Other Types of Skewed distributions

Gumbel Model

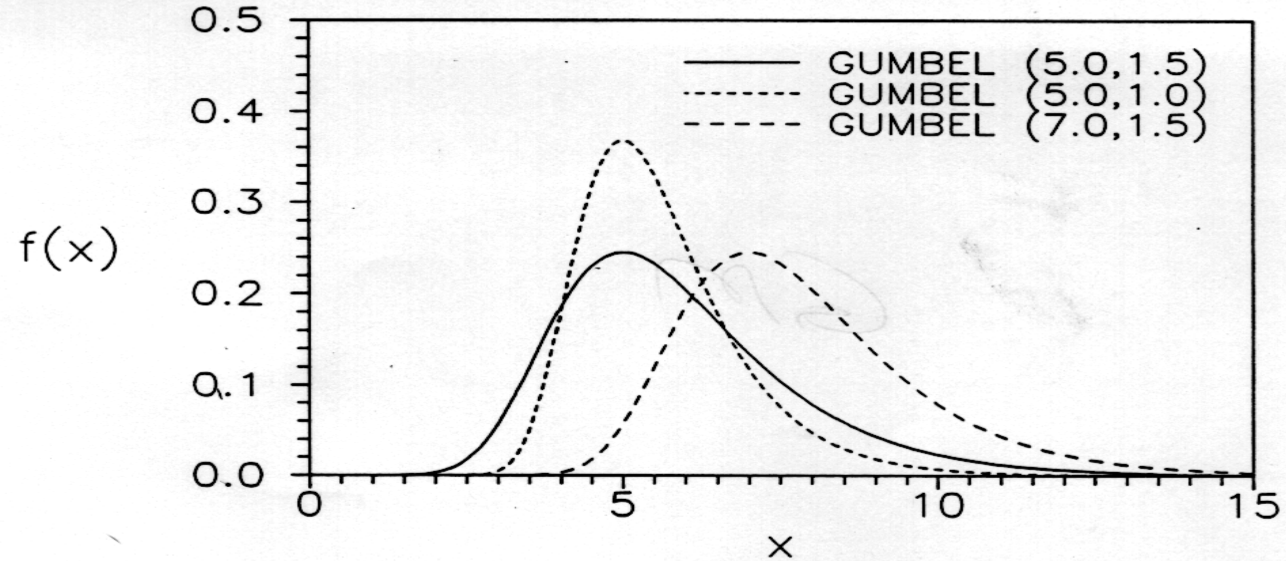


Fig. 4.18 **PDFs of the Gumbel model of Eq. (4.214) with parameters (a) $x_o = 5.0$, $\alpha = 1.5$; (b) $x_o = 5.0$, $\alpha = 1.0$; and (c) $x_o = 7.0$, $\alpha = 1.5$.**

Frequency Distribution Models

7 - Other Types of Skewed distributions

Log-Gumbel Model

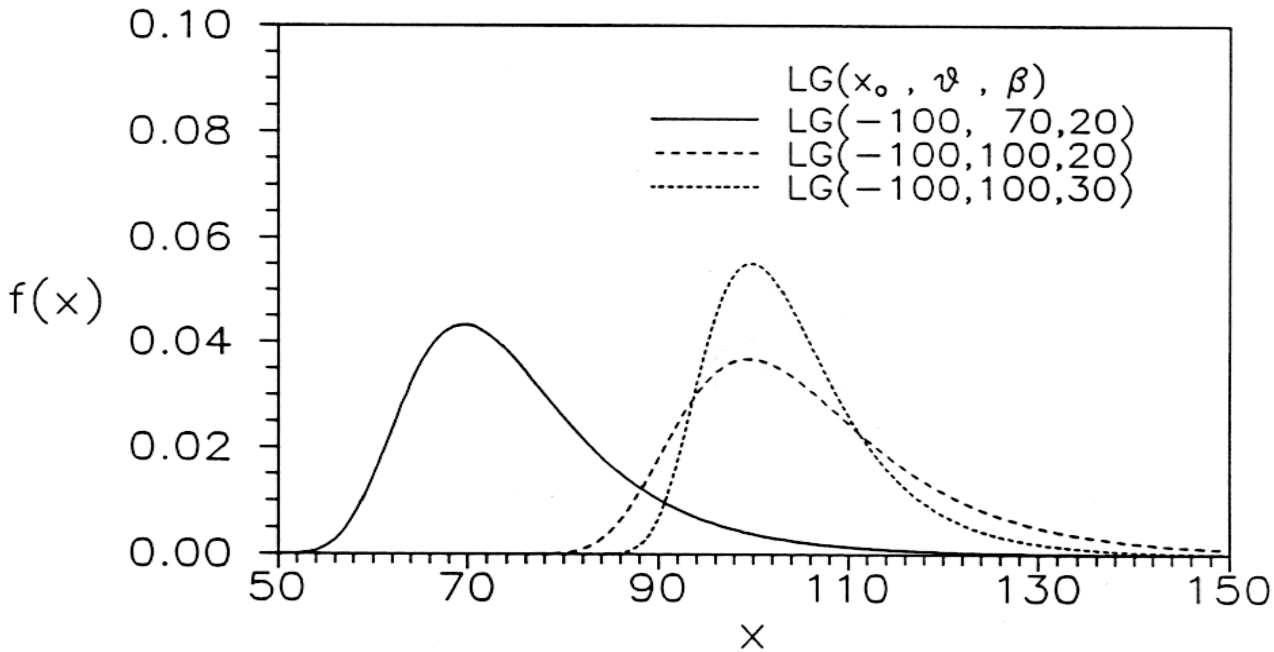


Fig. 4.20 **PDFs of the log-Gumbel model of Eq. (4.237) with parameters (a) x_o = -100, θ = 70, β = 20; (b) x_o = -100, θ = 100, β = 20; and (c) x_o = -100, θ = 100, β = 30.**

Frequency Distribution Models

7 - Other Types of Skewed distributions

Weibull Model

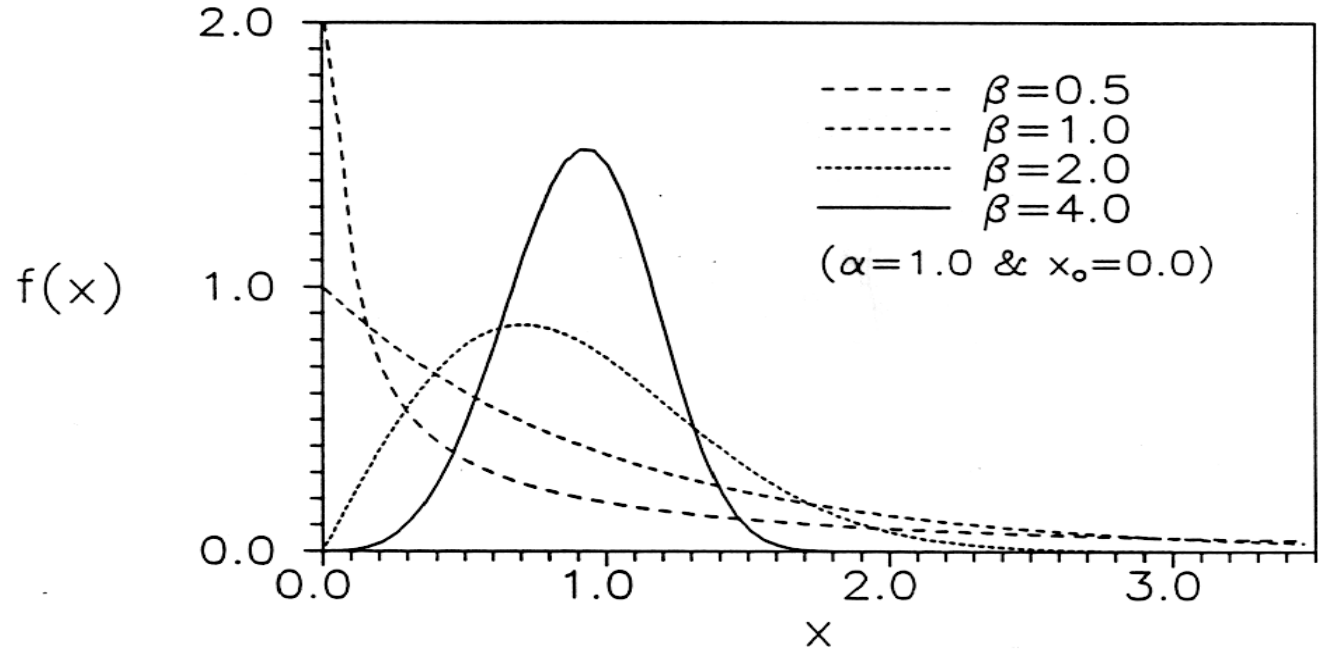


Fig. 4.22 **PDF of the three-parameter Weibull model of Eq. (4.264) with parameters x_o = 0, α = 1, and (a) β = 0.5, (b) β = 1.0, (c) β = 2.0 and (d) β = 4.0.**

Frequency Distribution Models

7 - Other Types of Skewed distributions

Beta Model

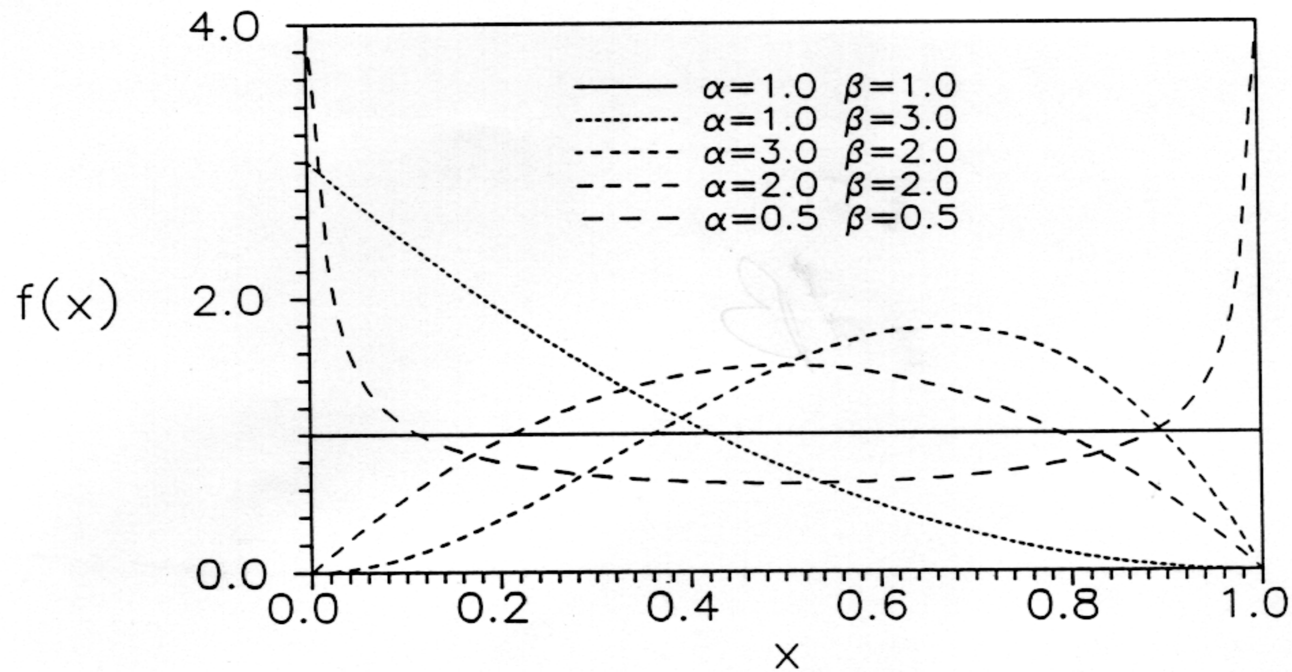


Fig. 4.24 Typical shapes of the PDF of the beta distribution of Eq. (4.293) for various sets of parameters.

Frequency Distribution Models

7 - Other Types of Skewed distributions

Wakeby Model

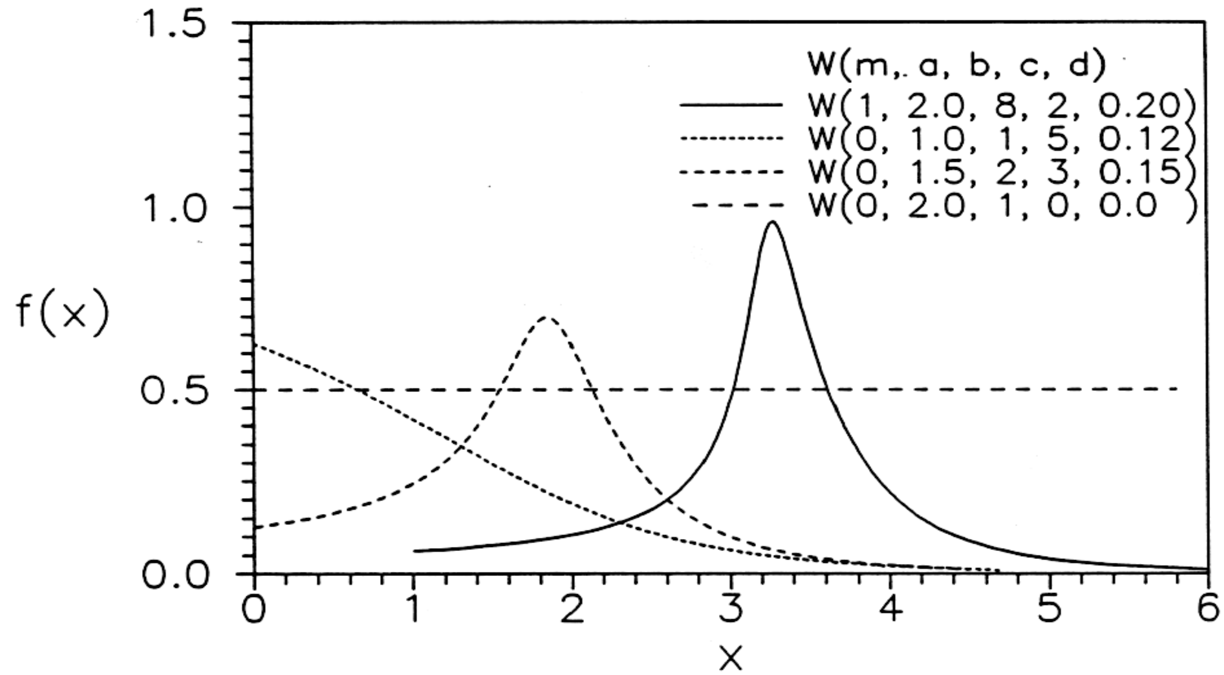


Fig. 4.25 PDFs of the Wakeby distribution for various sets of parameters.