Frequency Distribution Models

Probability Density Function (PDF)

- **What is a PDF model?**

  A mathematical equation that describes the frequency curve or probability distribution of a data set.

- **Why modeling?**

  – It represents and summarizes the statistical distribution of the entire "geologic phenomenon" and helps in making predictions.

  – Statistical characteristics and parameters can be derived and calculated easily from the model. Such parameters reflect the behavior of the "geologic phenomenon".

Frequency Distribution Models

Probability Density Function (PDF)

- **Types of models**

  – **Normal distribution**

    § Gaussian (Gauss), Laplace, or Bell-shaped distribution.

    § Values are symmetrically distributed around a central value.

    § The mean is the most representative value of the distribution (i.e. use arithmetic average to estimate the unknown from a set of "uncorrelated random variables").

    § The variance is the well-defined measure of the spread of observations.

Frequency Distribution Models

Probability Density Function (PDF)

Normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right]$$

$$F(x_0) = \int_{-\infty}^{x_0} f(x)dx$$

Frequency Distribution Models

Probability Density Function (PDF)

- **Types of models**

  - **Non-normal distribution**

    - § Lognormal distributions (natural or base-10 logarithms of measured observations).
    - § Values are asymmetrically distributed around a central value.
    - § The mean is not the most representative value of the distribution. (i.e. do not simply use arithmetic average to estimate the unknown from a set of "uncorrelated random variables"). However, other averages or median value might work!

Frequency Distribution Models

Probability Density Function (PDF)

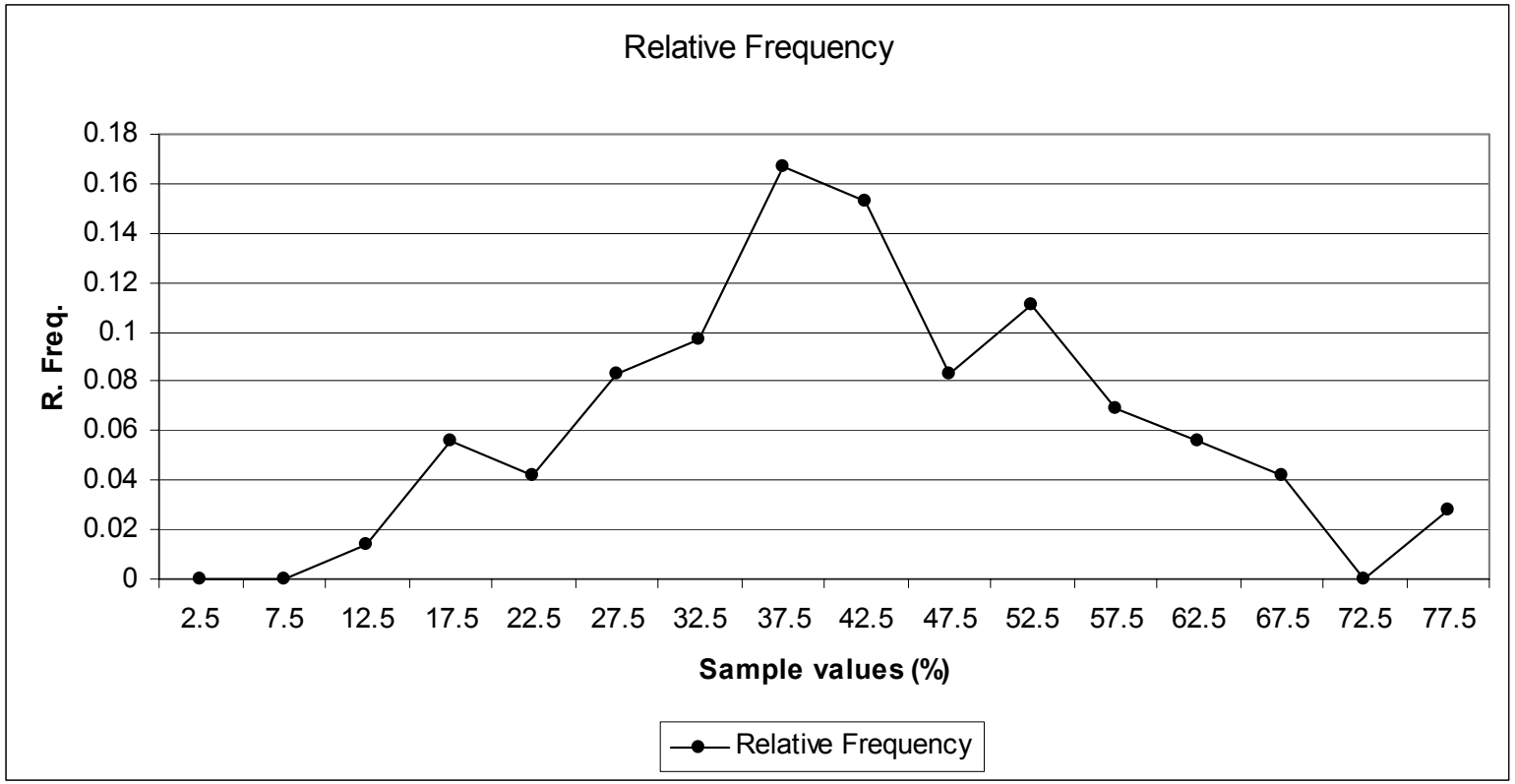
Non-normal distribution

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln \gamma - \ln x}{\beta} \right)^2 \right]$$

$$\gamma = e^{\alpha} \qquad \beta = S_{\log aritms}$$

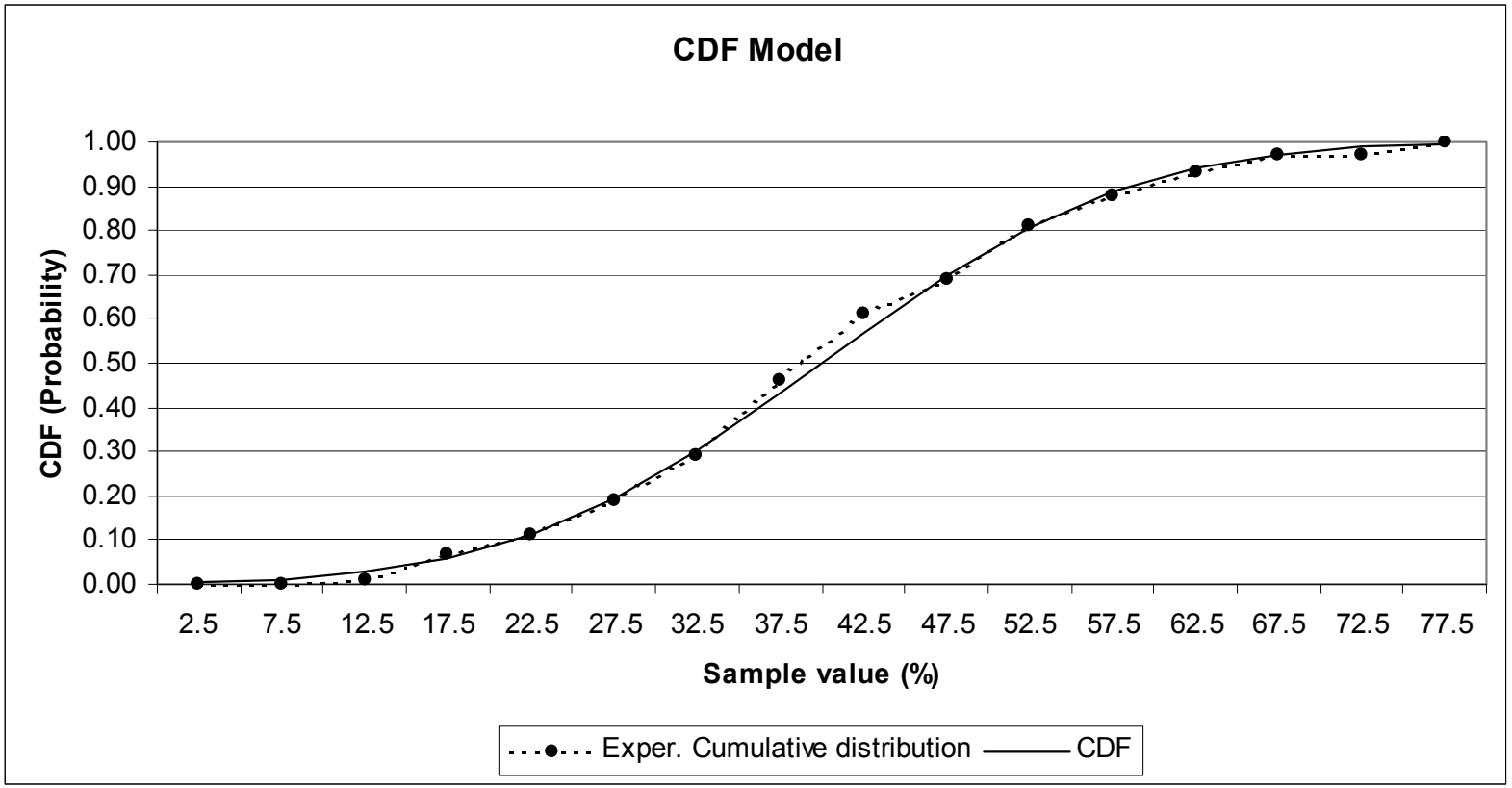
Frequency Distribution Models

Probability Density Function (PDF): Example of a normal case



Frequency Distribution Models

Cumulative Distribution Function (CDF): Example Example of a normal case



Frequency Distribution Models

Skewed Distributions

- **What if a distribution is skewed?**

  Use power transformation techniques to transform original data values into a defined power function.

- **Why and how?**

  – Distribution of transformed data is much easier to describe than the distribution of original skewed data.

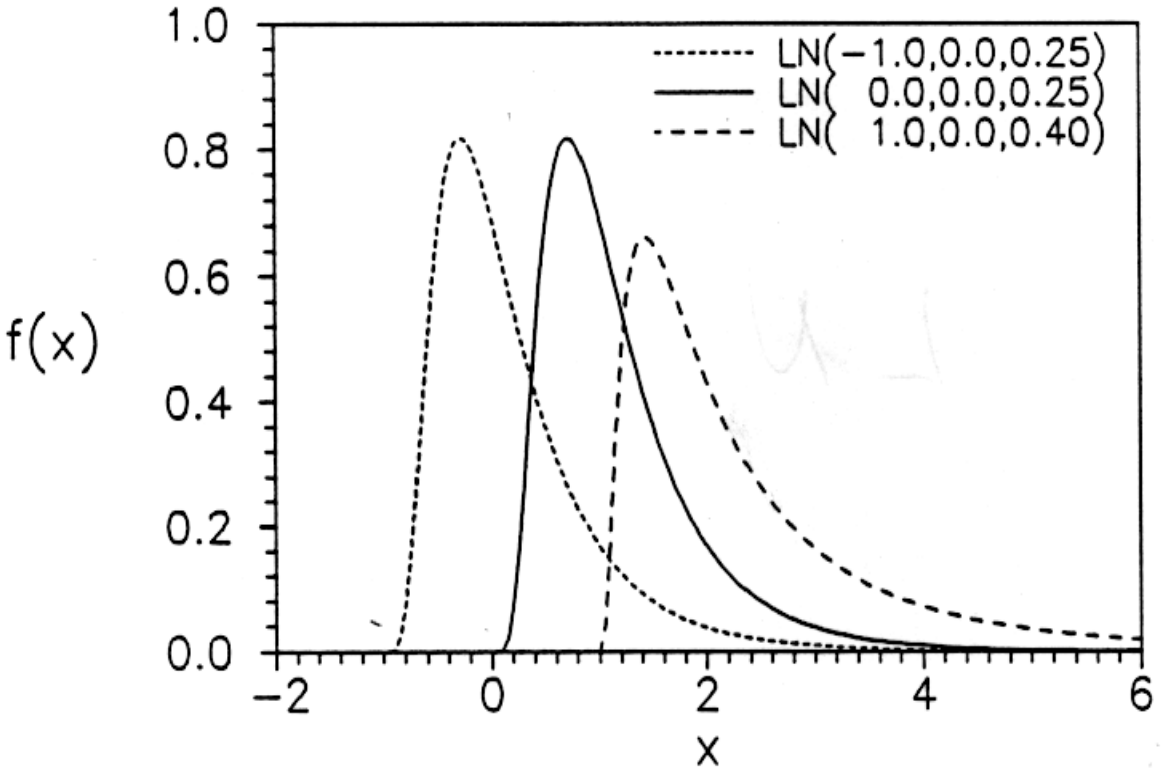
  – Common power transforms:

$$y = (z^p - 1) / p \qquad y = Ln(z)$$

Frequency Distribution Models

Other Types of Skewed distributions

Lognormal Model



Frequency Distribution Models

Other Types of Skewed distributions

Exponential Model

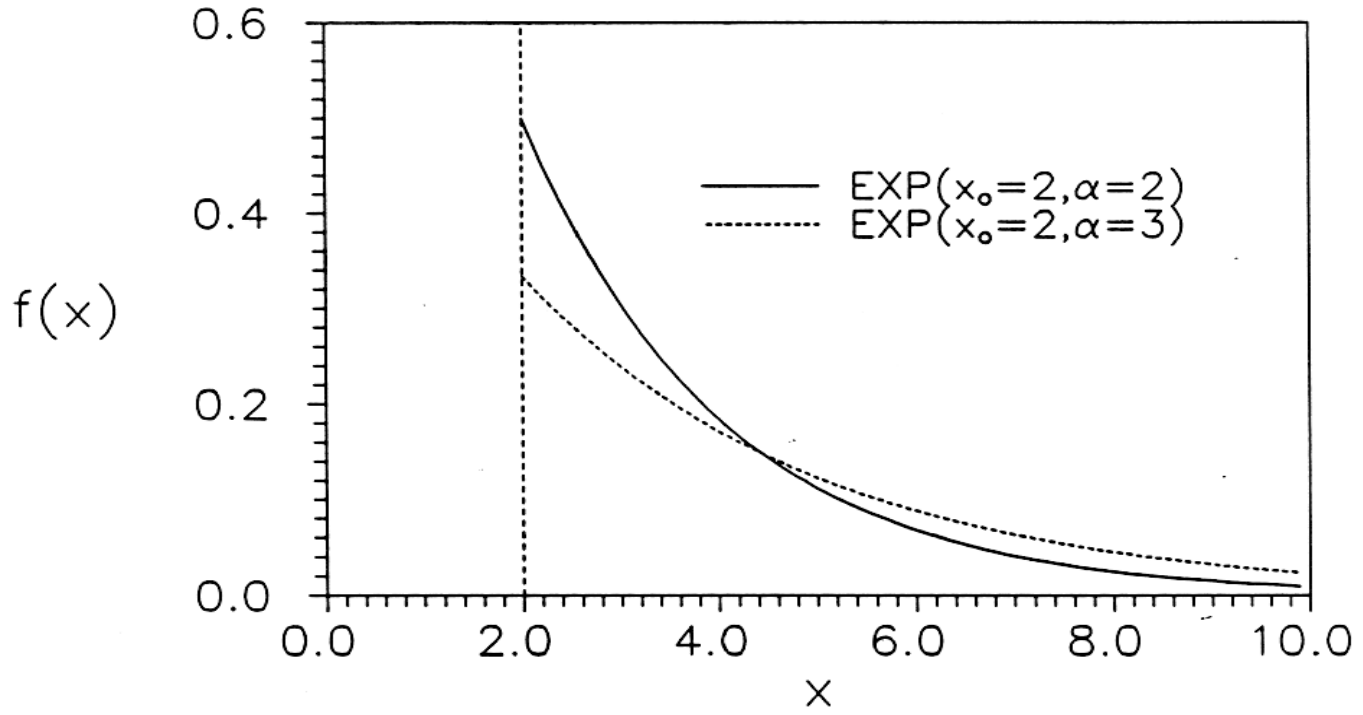
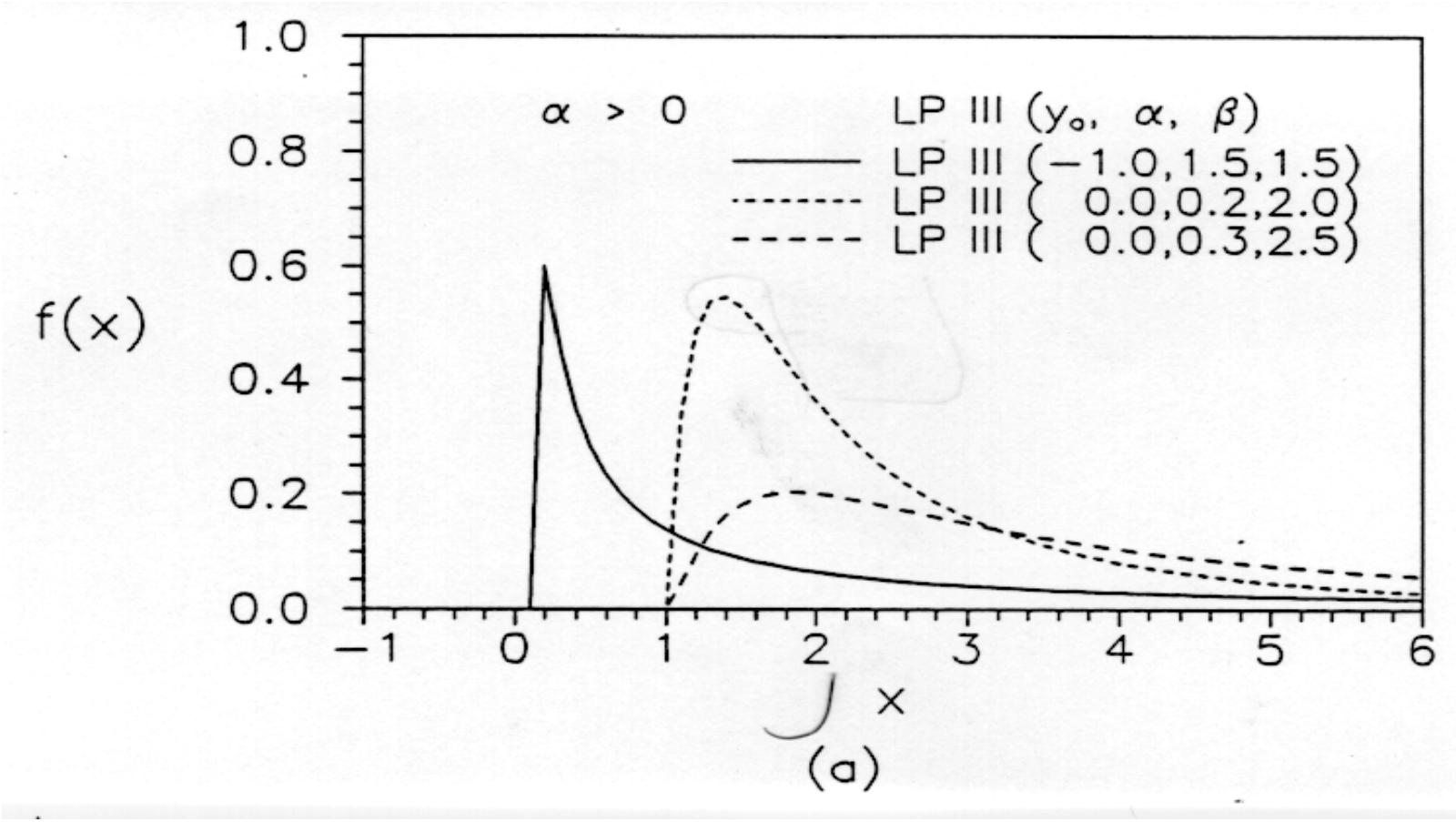


Fig. 4.7 **Examples of exponential density functions with parameters (a) $x_o = 2$, $\alpha = 2$ and (b) $x_o = 2$ and $\alpha = 3$.**

Frequency Distribution Models

Other Types of Skewed distributions

Log-Pearson Type III Model



Frequency Distribution Models

Other Types of Skewed distributions

General Extreme Value (GEV) Model

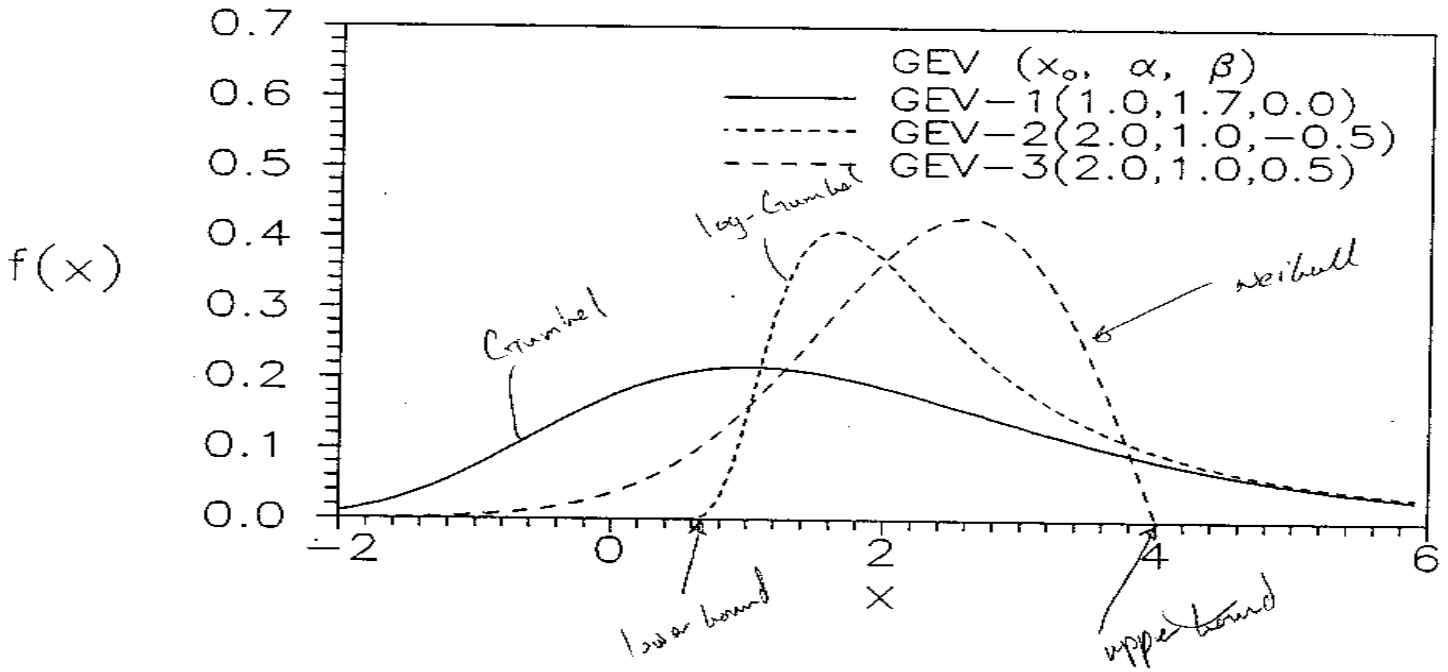


Fig. 4.14 GEV probability density functions of Eq. (4.188) with parameters (a) $x_o = 1$, $\alpha = 1.7$ and $\beta = 0$ (GEV-1); (b) $x_o = 2$, $\alpha = 1$ and $\beta = -0.5$ (GEV-2); and (c) $x_o = 2$, $\alpha = 1$ and $\beta = 0.5$ (GEV-3).

Frequency Distribution Models

Other Types of Skewed distributions

Gumbel Model

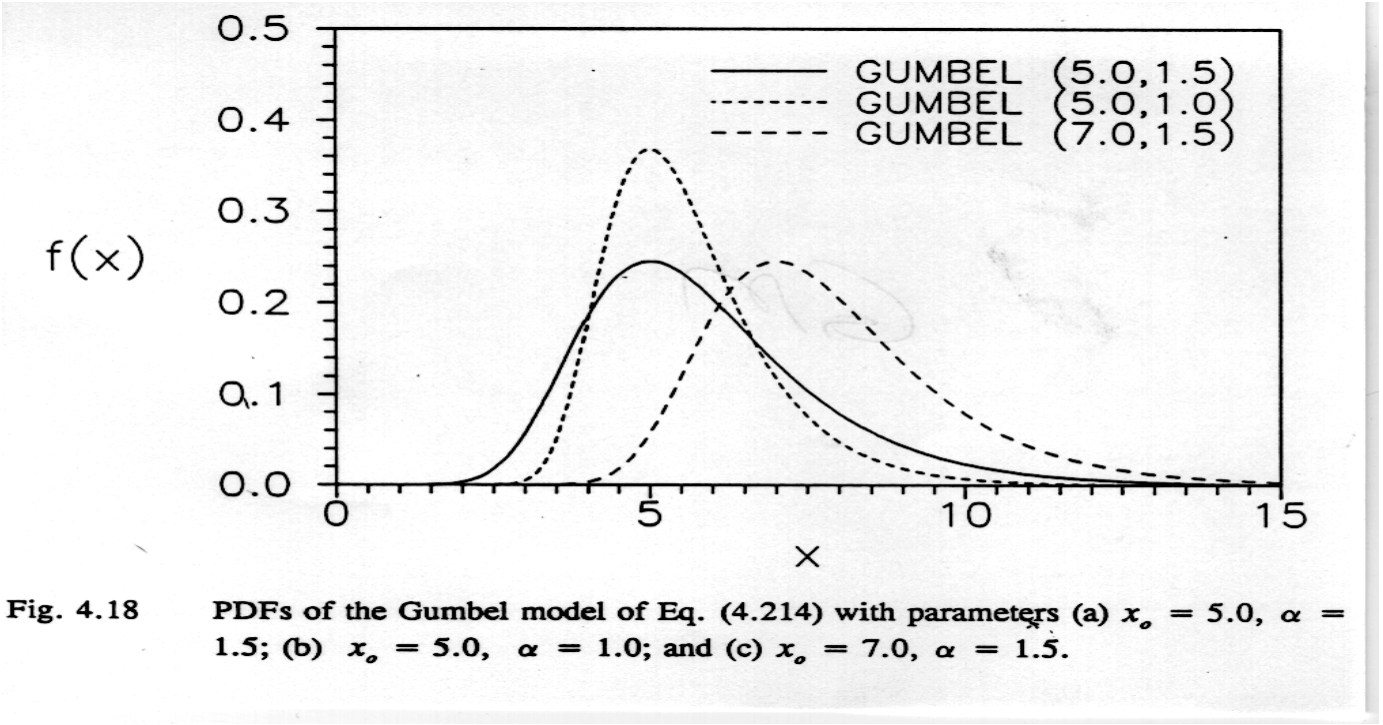


Fig. 4.18 PDFs of the Gumbel model of Eq. (4.214) with parameters (a) $x_o = 5.0$, $\alpha = 1.5$; (b) $x_o = 5.0$, $\alpha = 1.0$; and (c) $x_o = 7.0$, $\alpha = 1.5$.

Frequency Distribution Models

Other Types of Skewed distributions

Log-Gumbel Model

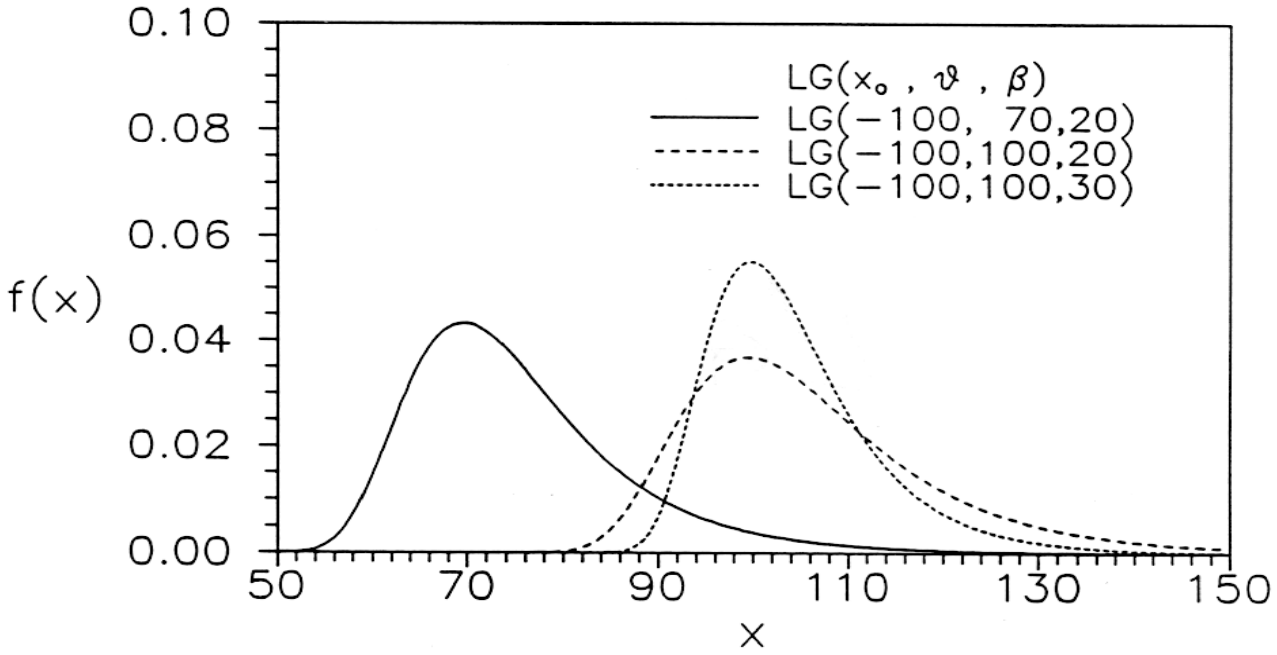


Fig. 4.20 PDFs of the log-Gumbel model of Eq. (4.237) with parameters (a) x_o = -100, θ = 70, β = 20; (b) x_o = -100, θ = 100, β = 20; and (c) x_o = -100, θ = 100, β = 30.

Frequency Distribution Models

Other Types of Skewed distributions

Weibull Model

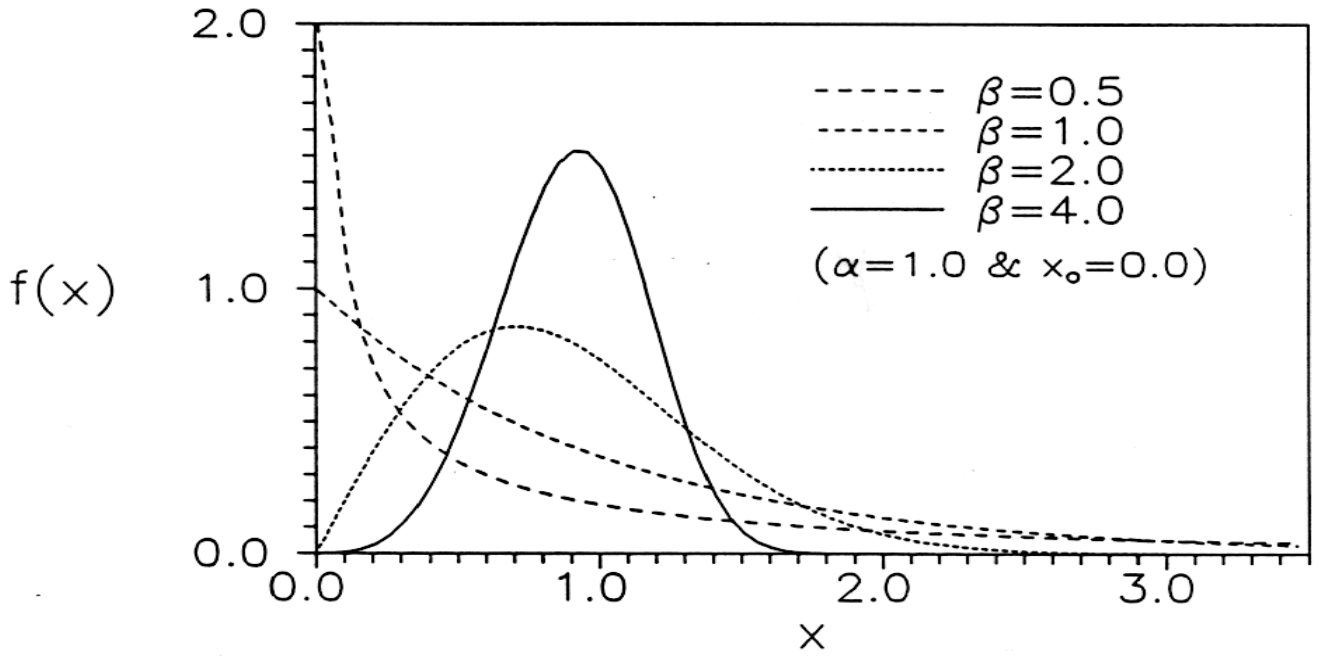


Fig. 4.22 PDF of the three-parameter Weibull model of Eq. (4.264) with parameters x_o = 0, α = 1, and (a) β = 0.5, (b) β = 1.0, (c) β = 2.0 and (d) β = 4.0.

Frequency Distribution Models

Other Types of Skewed distributions

Beta Model

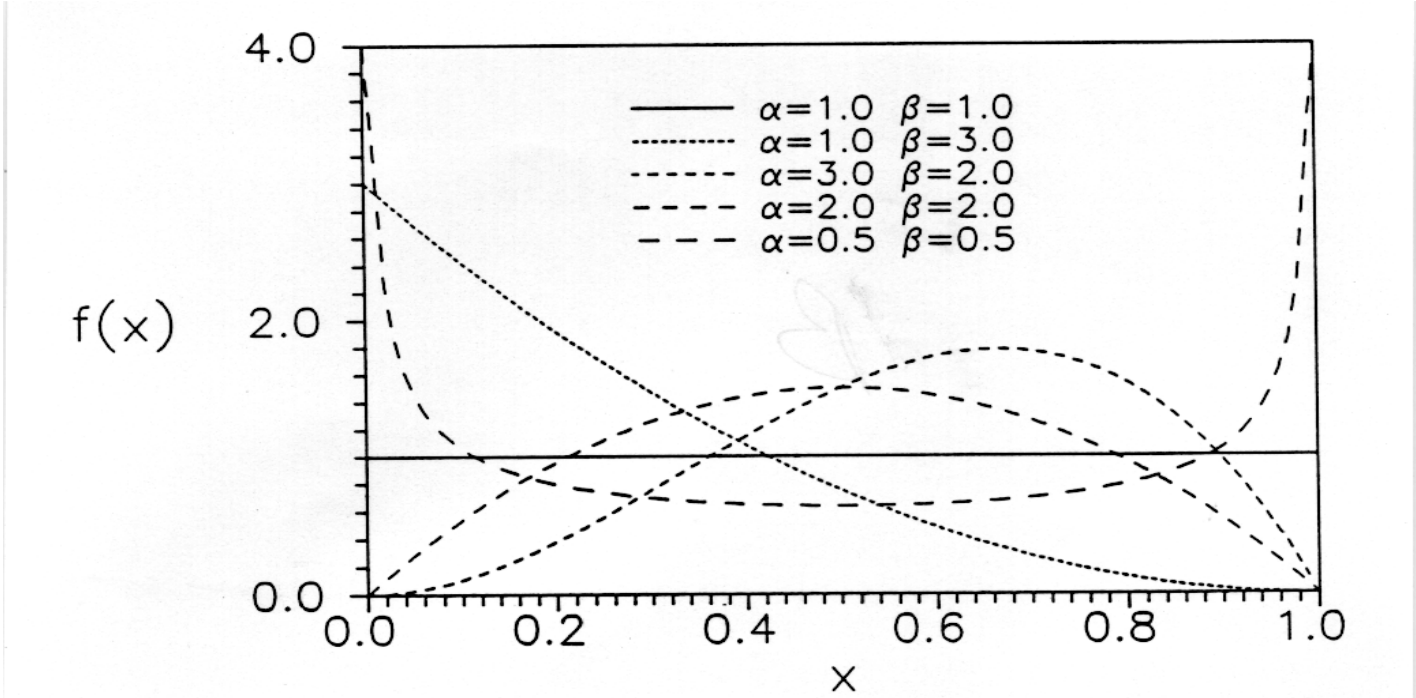


Fig. 4.24 Typical shapes of the PDF of the beta distribution of Eq. (4.293) for various sets of parameters.

Frequency Distribution Models

Other Types of Skewed distributions

Wakeby Model

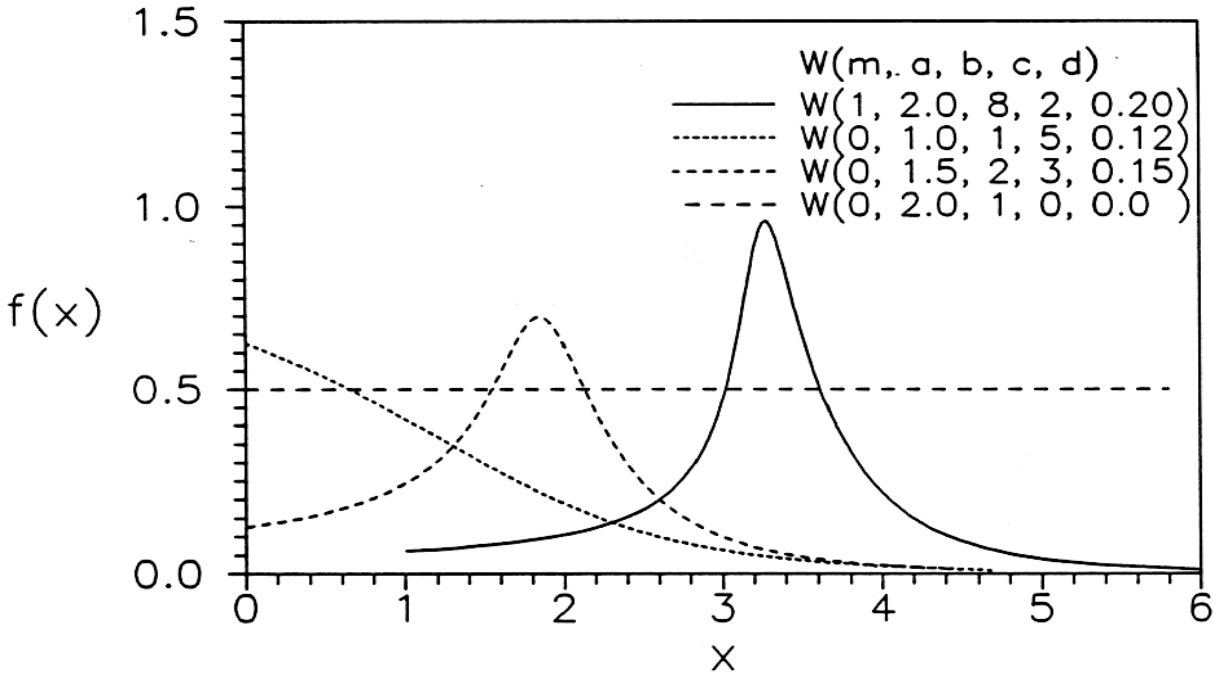


Fig. 4.25 **PDFs of the Wakeby distribution for various sets of parameters.**