EE 577: Wireless and Personal Communications

Chapter 06: Digital Modulation

What is Modulation?

- Modulation is the process of converting a baseband signal to a bandpass signal.
- A high frequency "carrier" is modulated by the original "modulating" signal to give a "modulated" wave.
- Why not propagate the baseband signal?
  - Baseband signals are not suitable for propagation
  - Antenna dimensions would be impractical
  - We can't all use the same low frequency spectrum at the same time

Analog vs. Digital Modulation

- The primary difference between analog and digital modulation is the source information
- If the source is analog, the modulation is analog.
- If the source is digital, the modulation is digital.
- The carrier is always analog

Analog Modulation Techniques

- Amplitude Modulation (AM)
- Angle Modulation
    - Frequency Modulation (FM)
    - Phase Modulation (PM)
- Angle Modulation is more resistant to noise than AM
- AM occupies less bandwidth

Angle Modulation

- **Two types:** Frequency Modulation and Phase Modulation

- Both types have the general form for the transmitted signal, $s(t)$:

$$s(t) = A_c \cos(\Theta(t))$$

- Here, $\Theta(t)$ is the total phase and it will be a function of the data message signal, $m(t)$

Phase Equation

- **FM:**

$$\Theta(t) = 2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau$$

- **PM:**

$$\Theta(t) = 2\pi f_c t + 2\pi k_p m(t)$$

Modulation Index

- **FM:** $\beta_{FM} = k_f A_m / W = \Delta f / W$
- **PM:** $\beta_{PM} = k_p A_m$

where:

A_m is the amplitude of the message

W is the message bandwidth

Δf is the max frequency deviation

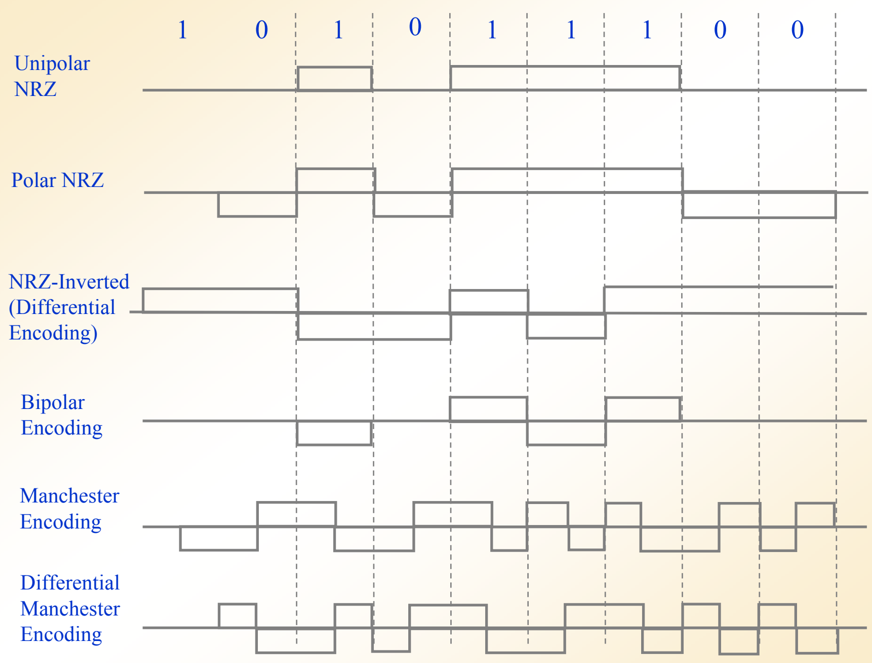
Analog Modulation

- The FM and PM spectrum is composed of an infinite series of discrete lines.
- The transmitted signal is not naturally bandlimited.
- Carson's Rule is frequently used to estimate the transmission bandwidth:

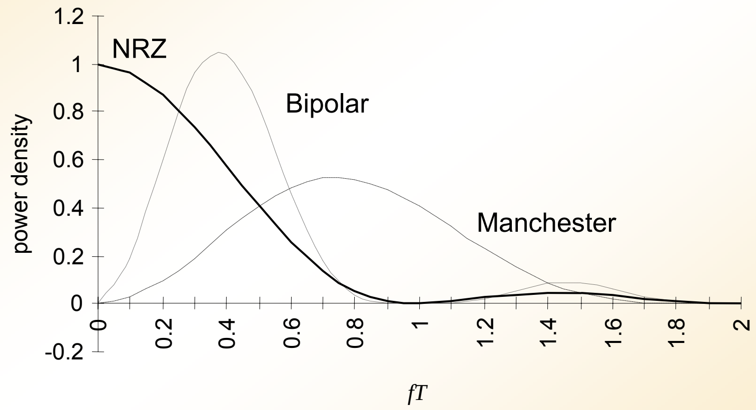
$$B_T = 2(\beta_f + 1)W$$

Digital Baseband Transmission (Line Coding)

- Method of converting a binary sequence into a digital signal.
- Design considerations:
  - Synchronization (ease of extracting bit timing)
  - Average transmission power
  - Inherent error detection
  - Power spectral density properties



Power Spectral Density (PSD)



Channel Capacity

- **Shannon** showed that the channel capacity, (C in bps), is the max transmission rate and is a function of the transmission bandwidth, B, and the received SNR

$$C = B \log_2(1 + \text{SNR}), \qquad \text{(AWGN)}$$

- Modulation systems cannot reach this limit without using error-control coding

Bandwidth Definitions

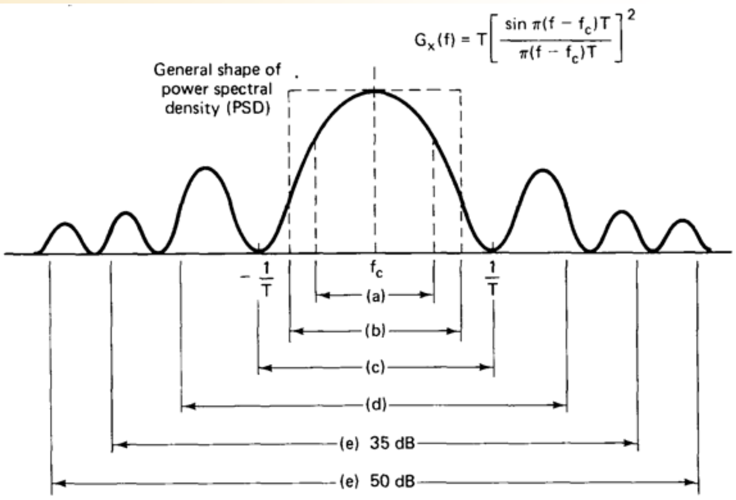


Figure 1.18 Bandwidth of digital data. (a) Half-power. (b) Noise equivalent. (c) Null to null. (d) 99% of power. (e) Bounded PSD (defines attenuation outside bandwidth) at 35 and 50 dB.

Pulse Shaping

- Baseband rectangular pulses are infinite-bandwidth signals
- The receiver and channel have finite bandwidths (finite bandwidth filters)
- High frequencies in the signal will not pass through the channel => symbol will be broaden in time (time-dispersion)
- The transmitted symbol will affect adjacent symbols on either side. This is called **I**nter **S**ymbol **I**nterference (ISI)

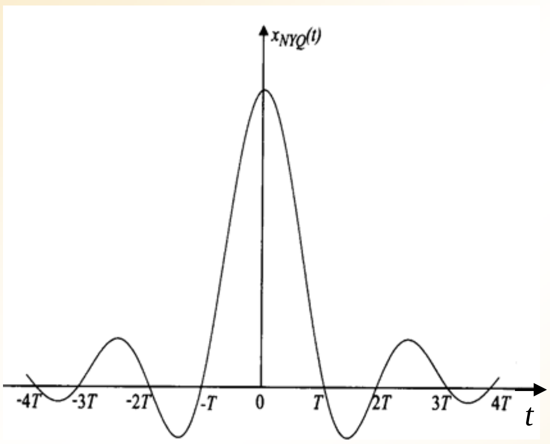
Nyquist Criterion

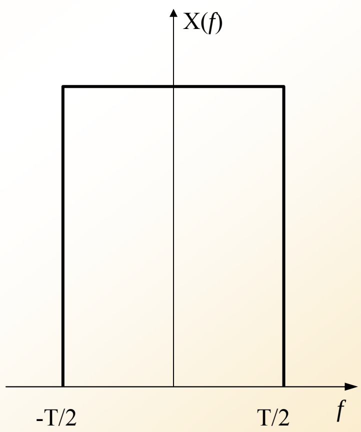
- **Nyquist** showed that the ISI can be removed if pulses have zero values for at all sampling instances except the current symbol
- This happens if the transmission acts like a filter with an impulse response:

$$h(nT_s) = K \quad \text{for } n = 0$$
$$= 0 \quad \text{for } n \neq 0$$

- Satisfied by a sinc pulse:
  - Unrealizable
  - Slow decay of the pulse in time (proportional to $1/t$)

Sinc Pulse





Raised-Cosine Pulse

The Raised Cosine Filter fulfills the Nyquist criteria:

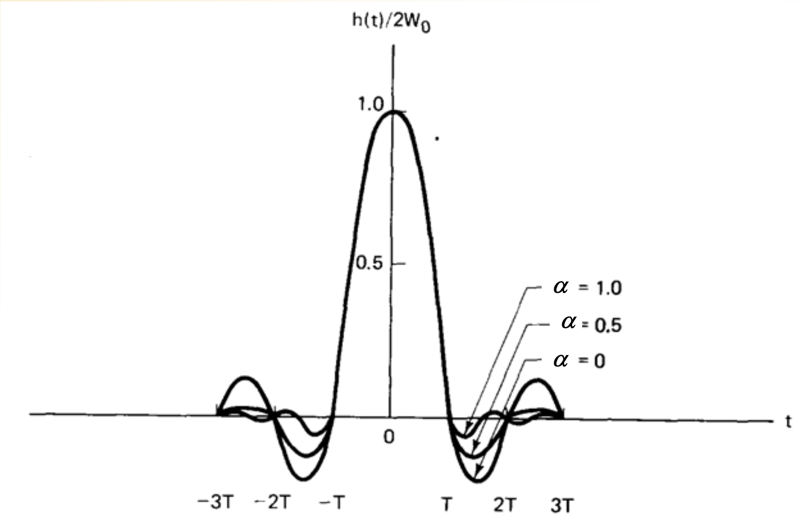
- "Half" of the filter is in the transmitter and "half" in the receiver giving rise to the "Square Root Raised Cosine"
- Receivers process several signals in a block to account for filter effects
- It expands the required bandwidth (by αW) at the cost of faster decay rate
- It is realizable

Raised-Cosine Pulse

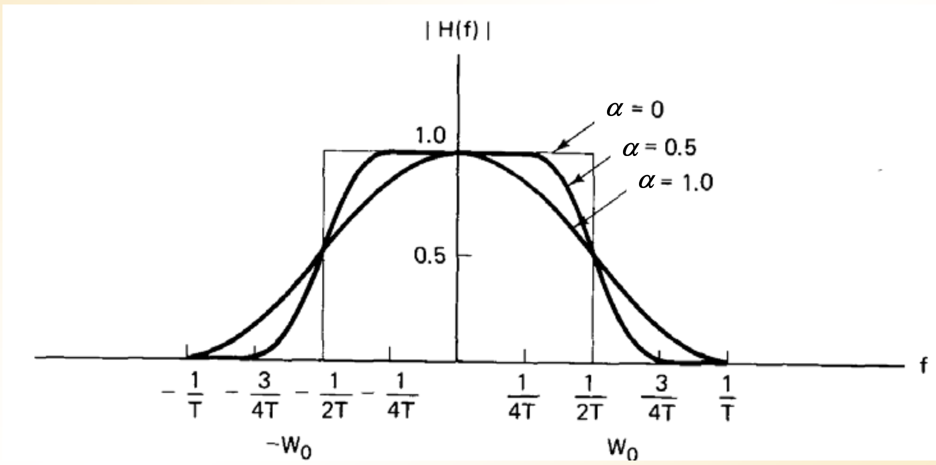
- Filter **Transfer Function**:
  - Pass band: $H(f) = T_S$
  - Transition: $H(f) = \frac{T_S}{2}\left[1 + \cos\left(\frac{\pi |f| - 1/(2T_S) + \alpha}{2\alpha}\right)\right]$
  - Stop band: $H(f) = 0$
- Filter **Impulse Response**:

$$h(t) = \left[\frac{\cos 2\pi\alpha t}{1-(4\alpha t)^2}\right]\left(\frac{\sin(\pi t / T_S)}{\pi t / T_S}\right)$$

Raised-Cosine Pulse



Raised-Cosine Filter



Raised-Cosine Pulse

The symbol rate, R_S, that can be passed is a function of the filter bandwidth B and the roll-off factor, α

$$R_S = \frac{2B}{1+\alpha}$$

Gaussian Pulse

- Raised-cosine pulse shape has zero-crossings => sidelobes regeneration after nonlinear amplifications
- The Gaussian filter is a common choice because its robustness to nonlinear amplifiers
- It is **NOT** a Nyquist pulse => ISI
- Tradeoff ISI performance with BW

Gaussian Pulse

- Filter **Transfer Function**:

$$H(f) = \exp(-\alpha^2 f^2)$$

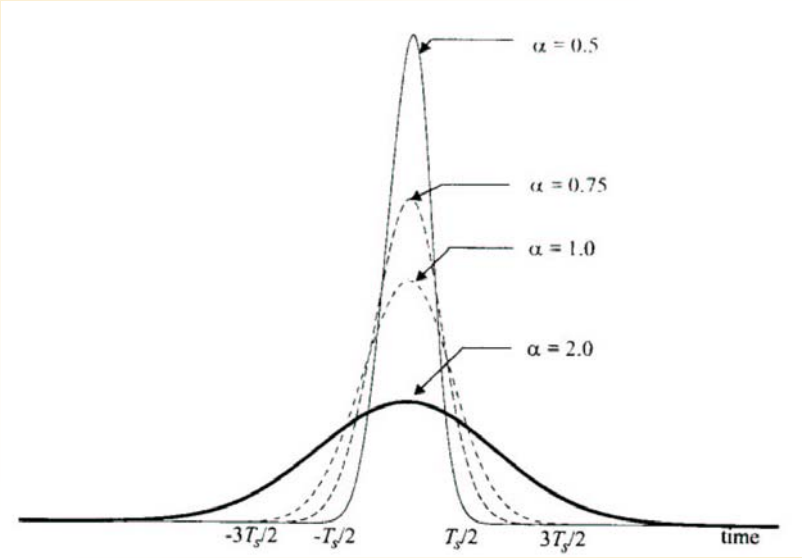
- Filter **Impulse Response**:

$$h(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} t^2\right)$$

$$\alpha = 0.5887/B$$

where B is the 3-dB BW of the filter

Gaussian Pulse



Digital Modulation

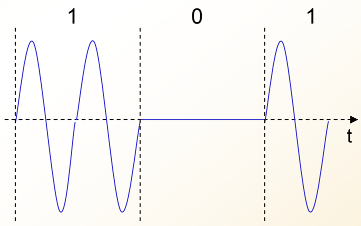
- The signal is sent on a carrier with a discrete set of possible signal levels
- Signals can be sent to represent a single bit at a time (binary) or to represent multiple bits at a time (M-ary)
- **Quality measures:**
  - Bit Error Rate (BER)
  - Power efficiency: E_b/N_0 for a fixed BER
  - Bandwidth efficiency: data rate per Hz of occupied bandwidth (bps/Hz)
  - Others: cost, complexity, interference, …

Digital Modulation Techniques

- Amplitude Shift Keying (ASK)
- Phase Shift Keying (PSK)
- Frequency Shift Keying (FSK)
- Hybrids

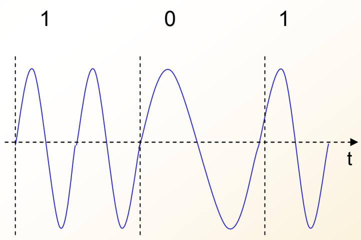
Amplitude Shift Keying (ASK)

- Very simple
- The digital source bits are represented by different amplitude levels
- Not commonly used alone in wireless



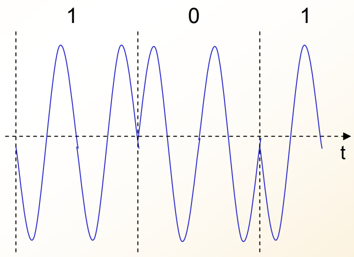
Frequency Shift Keying (FSK)

- The frequency of the carrier is switched depending on whether the source data is a "one" or "zero"
- Bandwidth depends on the distance between the carrier frequencies



Phase Shift Keying (PSK)

- The phase of the carrier is switched depending on whether the source data is a “one” or “zero”
- Robust against interference



Signal Constellations

- The **signal constellation** is a way of representing the signal as a phasor in a phase (signal) space without worrying about the carrier frequency

- The transmitted signal is represented by expanding it in terms of the basis functions of the space

- **Basis functions** are orthogonal functions (can not be expressed in terms of each others)

Signal Constellations

- The energy of basis functions is normalized to unity

  => Orthogonal + normalized = Orthonormal
- Each basis function represents a dimension of the signal space
- **Advantages:**
  - Signals can be viewed easily with respect to each other
  - Extract features of the modulation scheme
  - Can compute probability of error based on geometric arguments (e.g. distance)

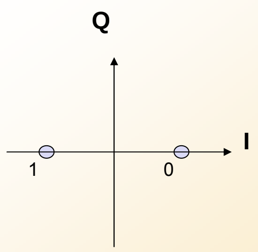
Signal Constellations

Example:

Binary Phase Shift Keying (BPSK) can be represented as drawing signals from the set:

$$\{ \sqrt{E_b}\, \phi_1(t), - \sqrt{E_b}\, \phi_1(t)\}$$

where $\phi_1(t) = (2/T_b)^{1/2} \cos(2\pi f_c t)$



Signal Constellation of BPSK

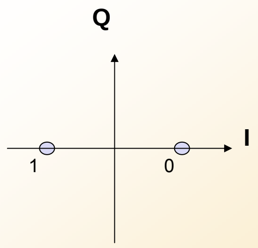
Phase Shift Keying (PSK)

- The Phase Shift Keying (PSK) scheme is popular for digital transmission
- Binary and Quadrature PSK are the most commonly used
- Coherent detection can be implemented at the receiver
- Non-coherent detection (using differential encoding) results in a ~ 3 dB performance penalty.

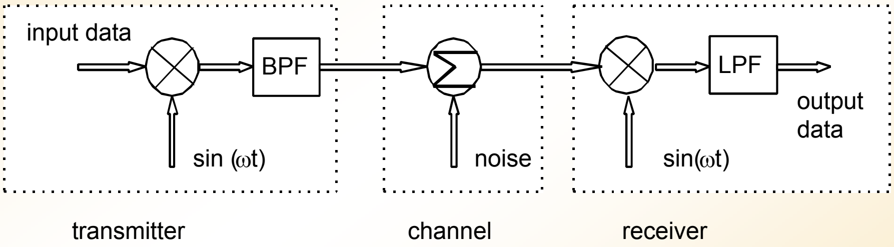
BPSK (Binary Phase Shift Keying)

- Bit value 0: cosine wave
- Bit value 1: inverted cosine wave (phase is π)
- Low spectral efficiency
- Robust, used e.g. in satellite systems
- Distance between the two signals is $2\sqrt{E_b}$

Signal Constellation of BPSK



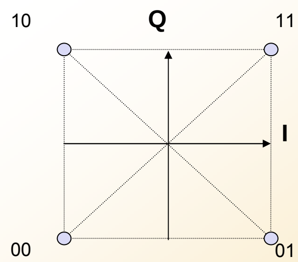
BPSK Modem Structure



QPSK (Quadrature PSK)

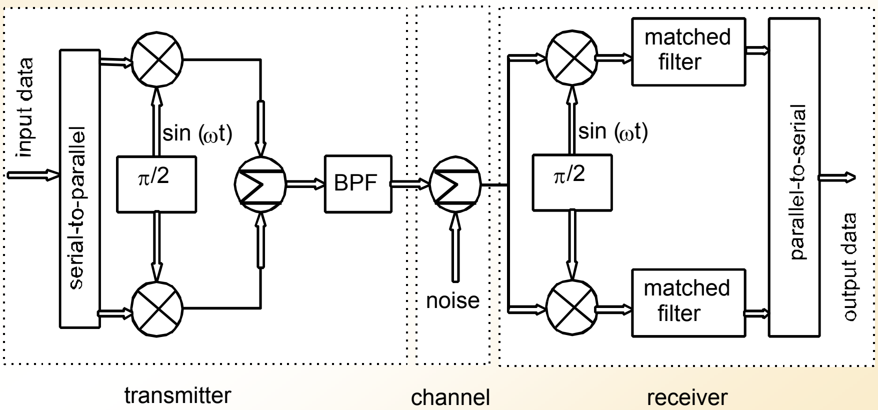
- 2 bits are coded as one symbol (signal)
- Modulated wave shifts between four phases, 90° apart to create signals for "00", "01" "10" or "11"
- Distance between the two signals is $2\sqrt{E_b}$

Signal Constellation of QPSK



QPSK Modem Structure

- Same performance as BPSK
- Double the BW efficiency

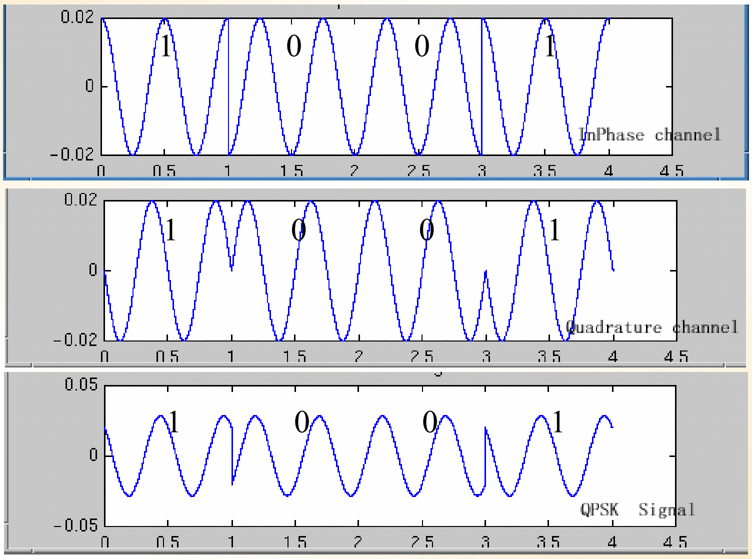


Example of QPSK

Input bits $0<t<T$	Phase of QPSK signal (radians)	Coordinates of signal Points S_{i1}	S_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

Example of QPSK

Input Binary Sequence: 1 1 0 0 0 0 1 1



Phase Shift Keying

Carrier representation:

- **BPSK:** $s(t) = d_i \cos(2\pi f_c t)$
- **QPSK:** $s(t) = d_i \cos(2\pi f_c t) + d_q \sin(2\pi f_c t)$

  where d_i and d_q are the individual data bits:
  - d_i is the "in phase" bit
  - d_q is the "quadrature phase" bit
  - d_i and d_q can come from either a single source or two sources
  - The latter with different data rates is possible

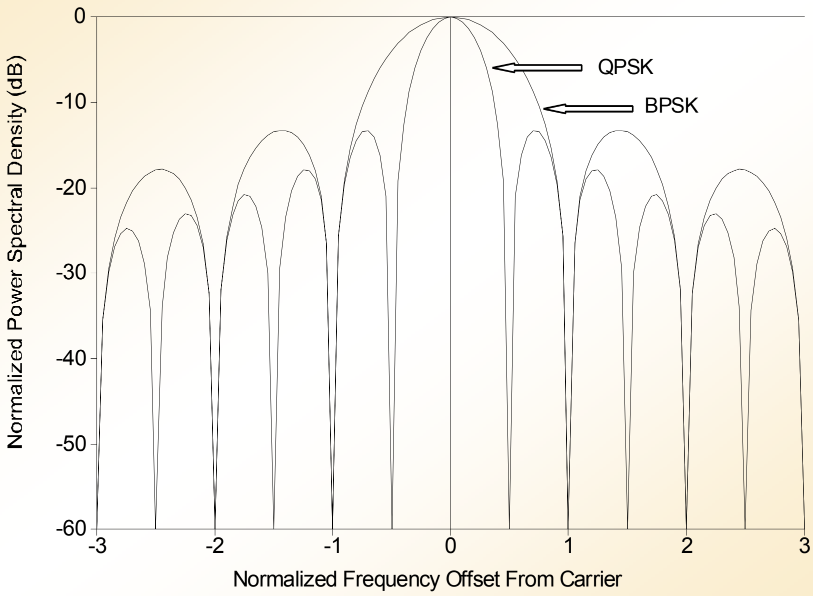
Power Spectral Density (PSD) of PSK

- It provides information about how power is distributed over frequencies
- The baseband PSD for PSK modulation:

$$\text{BPSK:} \quad P(\Delta f) = T_S \left\{ \frac{\sin(\pi \Delta f T_S)}{\pi \Delta f T_S} \right\}^2$$

$$\text{QPSK:} \quad P(\Delta f) = 2T_S \left\{ \frac{\sin(2\pi \Delta f T_S)}{2\pi \Delta f T_S} \right\}^2$$

PSD of PSK



Bandwidth of BPSK

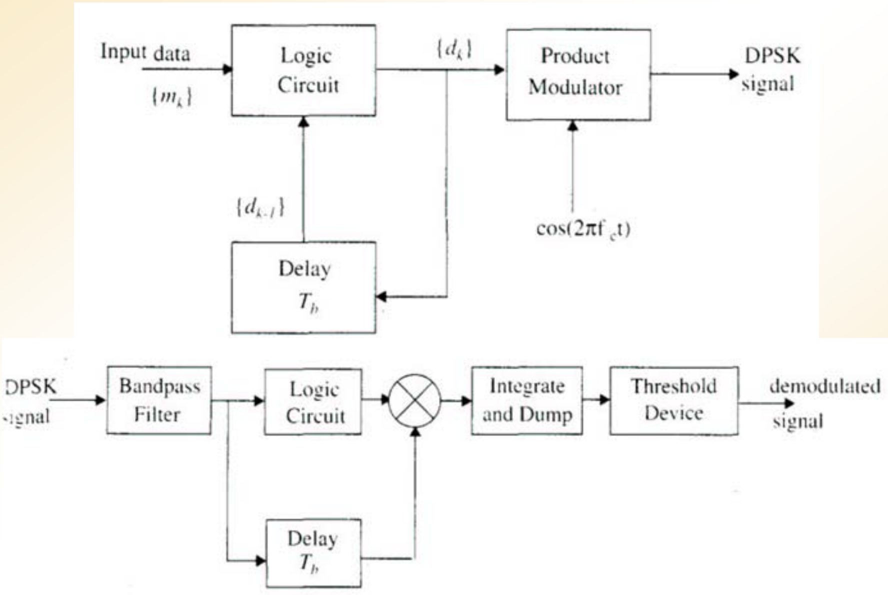
- For a rectangular pulse:
  - Null-null BW = $2R_b$
  - 90% BW = $1.6R_b$
- For a Raised-Cosine pulse with $\alpha = 0.5$:
  - 90% BW = $1.5R_b$
- For QPSK, BW is half that of BPSK

Differential PSK (DPSK)

- It is a noncoherent PSK scheme
- **Advantage:** easy implementation and cheap
- The modulating bit stream is the NOT the information stream
- Instead, it is a differentially encoded stream
- The modulating bit stream:

$$d_k = \text{COMP}\{m_k \otimes d_{k-1}\}$$

DPSK



Offset QPSK (Staggered QPSK)

- QPSK (after pulse shaping) do not have constant amplitude, especially for the transition with a phase shift equal to π (zero-crossing)

- Hence, non-linear power amplifiers cause distortion and create more spectral sidelobes (spectral re-generation)

- In offset QPSK, I and Q channels are offset in time by half a symbol duration (i.e., one bit duration)

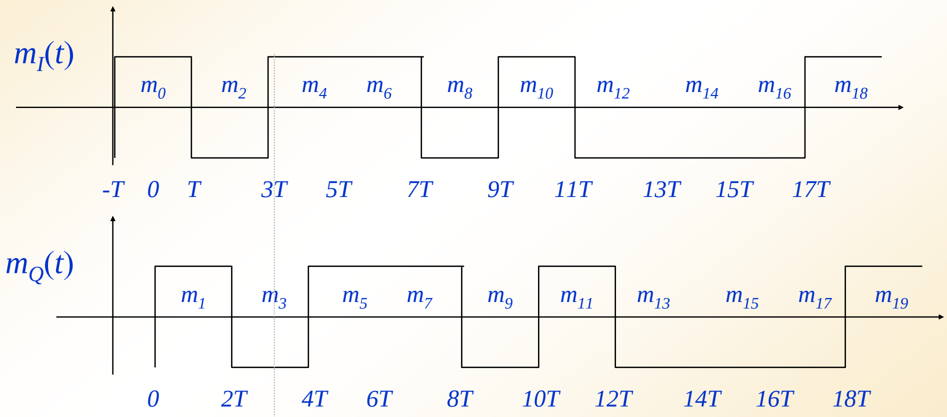
Offset QPSK

- At the transmitter side, shift one bit stream (in-phase or quadrature) by 1 bit duration (one-half the symbol duration)
- At the receiver side, shift the other to make them phase aligned
- Only one bit can change during each bit interval => signals can not go through an instantaneous ±180º phase shift
- In QPSK, ±180º phase shift can happen => bad with non-linear amplifiers

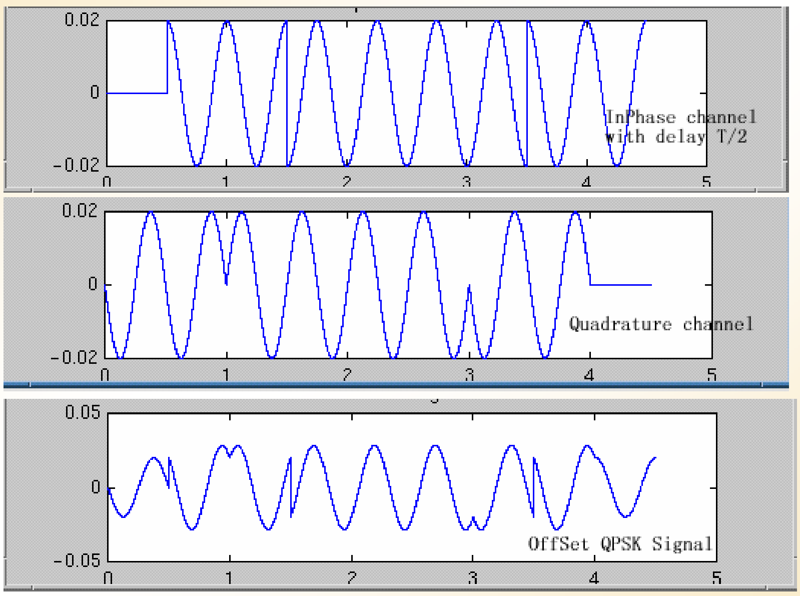
Offset QPSK

- Phase transitions happen at every half of a symbol period (instead of every symbol period as in QPSK), but each transition is limited to +/-90 degree
- **Result:** relatively more constant envelope after pulse shaping
- Hence, power-efficient non-linear amplifiers can be used without having distortion (or low spectral re-generation)

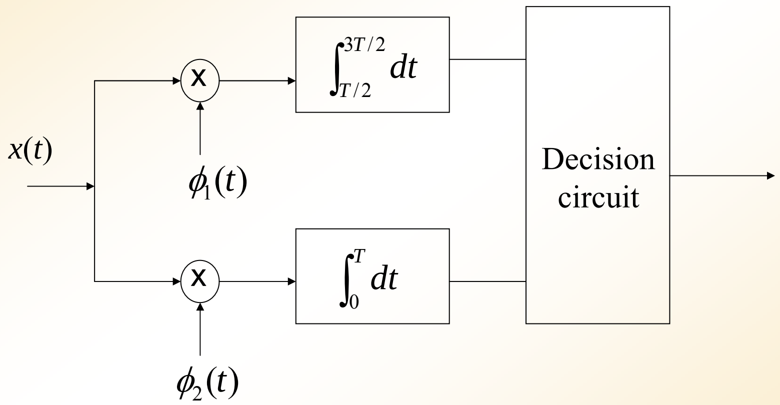
Offset QPSK



Example of Offset QPSK



Coherent Receiver of OQPSK

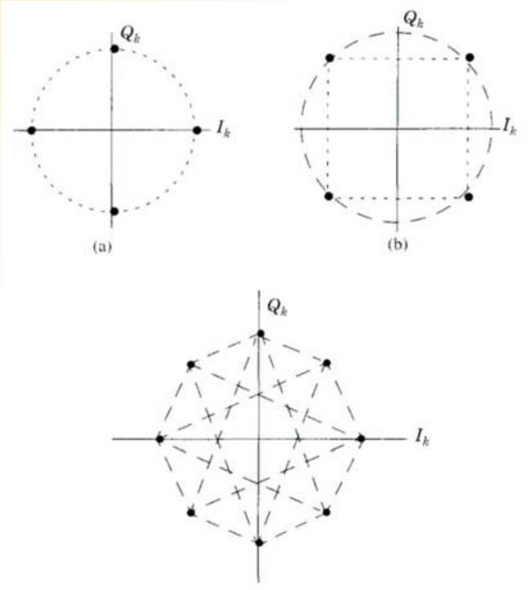


The detector makes a decision at time $3T_b/2$

π/4 - QPSK

- Compromise between OQPSK and QPSK:
  - Phase transitions can occur at $2nT_b$
  - Possible phase changes: ±π/4, ±3π/4
  - Maximum phase transitions limited to ±135° instead of ±180° (for QPSK) or ±90° (for OQPSK)
- Uses 2 QPSK constellations, one shifted by 45 degrees
- Alternates the use of each constellation with each successive symbol

π/4 - QPSK



π/4 - QPSK

- Prevents transition through the origin (±180° phase shift), so non-linear amplifiers can be used
- Ensures a phase shift of at least π/4 for each bit which helps in timing recovery

Symbol	Absolute phase value
1 1	0
-1 1	π/2
-1 -1	π
1 -1	- π/2

Symbol	Absolute phase value
1 1	π/4
-1 1	3π/4
-1 -1	-3π/4
1 -1	- π/4

Example: $\pi/4$ -QPSK

- **Information message is:** 1 1 0 1 0 1 0 0 1 1
- **Phase transition is:**

1 1	0 1	0 1	0 0	1 1
0	$3\pi/4$	$\pi/2$	$-3\pi/4$	0

Symbol	Absolute phase value
1 1	0
0 1	$\pi/2$
0 0	π
1 0	$-\pi/2$

Symbol	Absolute phase value
1 1	$\pi/4$
0 1	$3\pi/4$
0 0	$-3\pi/4$
1 0	$-\pi/4$

$\pi/4$ - QPSK

- **Advantage:**
  Performs better than OQPSK in the presence of multipath spread and fading

- **Disadvantage:**
  More sensitive to non-linear amplification than OQPSK

π/4–Differential QPSK (π/4–DQPSK)

- It is π/4–QPSK with differential encoding
- The data is differentially encoded by mapping from the data stream to the phase transtion according to the table:

$$\text{I channel : } \cos\ \theta_k$$

$$\text{Q channel : } \sin\ \theta_k$$

where $\theta_k = \theta_{k\text{-}1} + \phi_k$

Symbol	Phase transition ϕ_k
1 1	$\pi/4$
-1 1	$3\pi/4$
-1 -1	$-3\pi/4$
1 -1	$-\pi/4$

π/4–DQPSK

- **Advantage:**
  - Performs better than OQPSK in the presence of multipath spread and fading
  - Non-coherent detection can be used
- **Disadvantage:**
  - More sensitive to non-linear amplication

Example: π/4-DQPSK

- **Information message is:** 1 1 0 1 0 1 0 0 1 1
- **Phase transition is:** $\theta_k = \theta_{k-1} + f_k$

	1 1	0 1	0 1	0 0	1 1
0	π/4	π	π/4	-π	-3π/4

(initial phase is arbitary)

Symbol	Phase transition ϕ_k
1 1	$\pi/4$
0 1	$3\pi/4$
0 0	$-3\pi/4$
1 0	$-\pi/4$

M-ary Modulation

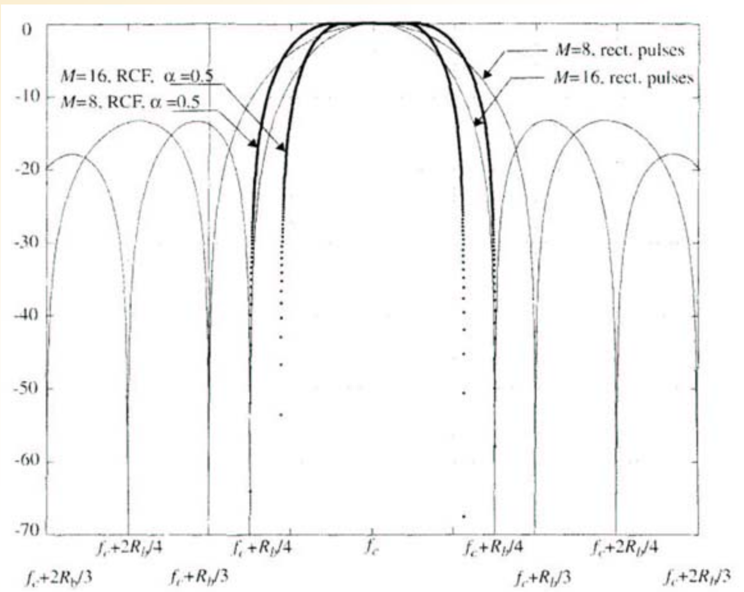
- One of M signals is transmitted according to the m-length bit sequence ($M = 2^m$)
- m bits are transmitted in each symbol interval
  => Higher BW efficiency
- However, signals are now closer in distance (in the signal constellation)
  => Higher probability of error

M-ary PSK (MPSK)

- A carrier signal is transmitted with one of M possible phases
- The M signals have different phases separated by $2\pi/M$
- Signal points are equally spaced on a circle of radius $\sqrt{E_s}$ from the origin
- Distance between adjacent signal points is:

$$2\sqrt{E_s}\sin(\pi/M)$$

PSD of MPSK



M-ary QAM (M-QAM)

- It combines phase and amplitude shift keying (amplitude is not constant)
- The *M* signals have different phases and amplitudes
- Signal points are placed on rectangles with increasing distance from the origin
- Distance between adjacent signal points is not constant
- Used with coherent detection and linear amplification (not suitable for wireless)

Frequency Shift Keying (FSK)

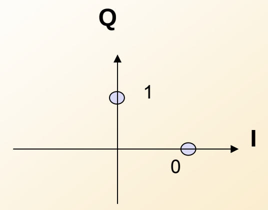
- FSK has the advantage of constant envelope transmission (signal amplitude remains constant)
- Suitable for nonlinear power amplifiers (high power efficiency)
- At the cost of larger occupied bandwidth
- Famous FSK schemes:
  - FSK
  - MFSK
  - MSK
  - GMSK

Binary FSK (BFSK)

- Bit 0: cosine wave with freq. f_1
- Bit 1: cosine wave with freq. f_2
- Frequencies are separated by Δf
- Frequencies should be orthogonal for the best performance

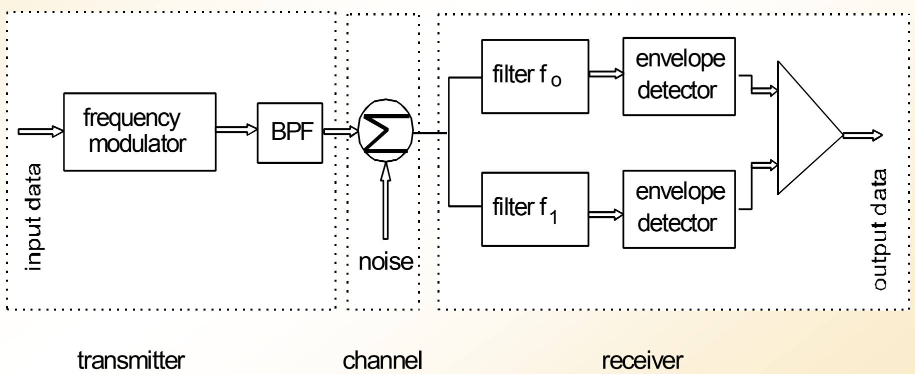
Condition: $\Delta f \geq 1/2T_b$

Signal Constellation of BFSK



BFSK Modem Structure

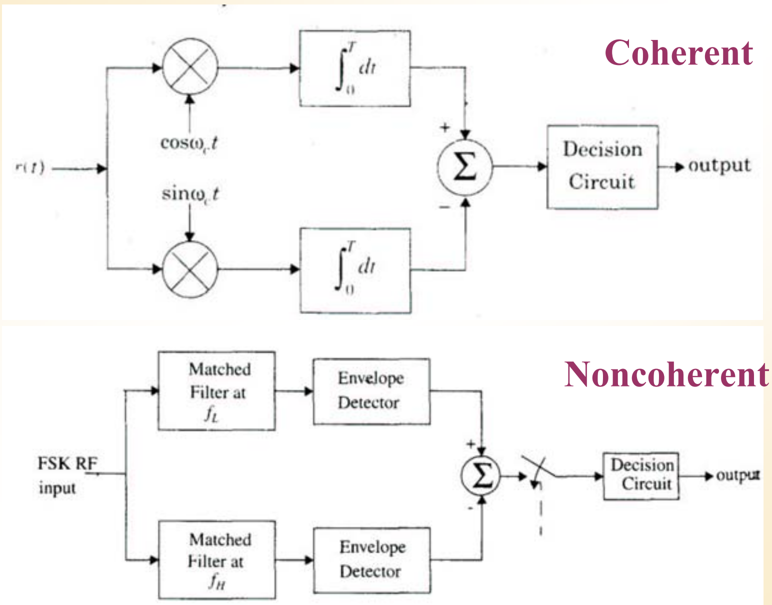
Generated by frequency modulating a carrier with a binary waveform (stream of pulses)



BFSK

- Distance between signals is $\sqrt{2E_b}$
  => needs 3 dB more SNR to achieve same BER as BPSK
- PSD falls at a rate proportional to Δf^{-4}
- BW = $2(\Delta f + R_b)$, rectangular pulse
  = $2[\Delta f + (1+\alpha)R_b]$, RC pulse
- Low spectral efficiency
- Noncoherent detection using envelop detection (or square-law combining) is possible
- Robust - used in wireless systems

Detectors for BFSK



M-ary FSK (MFSK)

- A cosine signal is transmitted with one of M possible frequencies
- Carrier frequencies are separated by $1/2T_s$
- Signal points are equally spaced on an M-dimensional space
- Distance between different signal points is $\sqrt{2E_s}$
- Noncoherent receiver consists of M matched filters (one for each frequency) followed by envelop detectors

Continuous Phase Modulation (CPM)

- The phase of the carrier varies continuously with time according to:

$$\Phi(t)=2\pi \sum_{k=-\infty}^{n} I_k h_k \int_0^t g(\tau - kT_s)d\tau, \qquad nT_s \le t \le (n+1)T_s$$

- The CPM signal is: $s(t) = A\cos(2\pi f_c t+\phi(t))$
- $\{I_k\}$ is M-ary information sequence
- $\{h_k\}$ are set of modulation indices. They can be the same (single-h) or different (multi-h)

CPM

Bit Energy

Carrier Frequency

$$s(t) = \sqrt{\frac{2E_b}{T}} \cos\left[2\pi f_c t + \Phi(t;I) + \theta_0\right]$$

Bit Duration

The CPM Phase

Initial Carrier Phase

CPM

M-ary Sequence

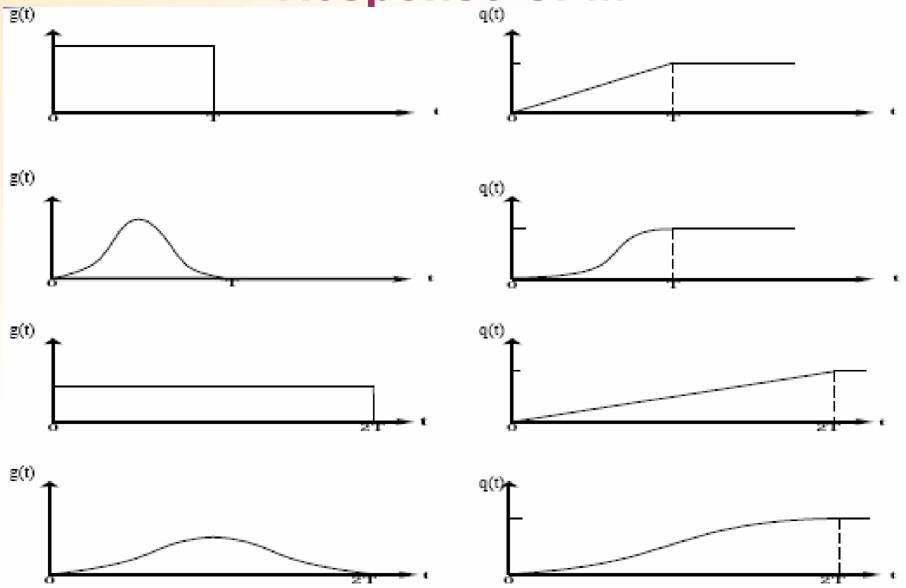
Modulation Index

$$\Phi(t;I) = 2\pi \sum_{k=-\infty}^{n} I_k h_k q(t - nT)$$

Pulse Shape

$$q(t) = \int_0^t g(\tau) d\tau$$

Pulse shapes for Full and Partial-Response CPM



CPM

- $g(t)$ is a pulse, e.g., rectangular, RC, …
  - If $g(t) = 0$, $t > T_s$ => Full-response CPM
  - If $g(t) \neq 0$, $t > T_s$ => Partial-response CPM
- Different CPM signals can be generated by varying M, $\{h_k\}$ and $g(t)$
- If $q(t)$ is rectangular pulse $0 \leq t \leq T_s$, the CPM is Continuous-Phase FSK (CPFSK)
- Furthermore, if $h_k = h = ½$ => CPFSK is called **Minimum Shift Keying (MSK)**

Minimum Shift Keying (MSK)

h = 0.5

$$s(t) = A\cos\left[2\pi\left(f_c + \frac{1}{4T}I_n\right)t - \frac{1}{2}n\pi I_n + \theta_n\right]$$

Binary

$$f_1 = f_c - \frac{1}{4T}$$

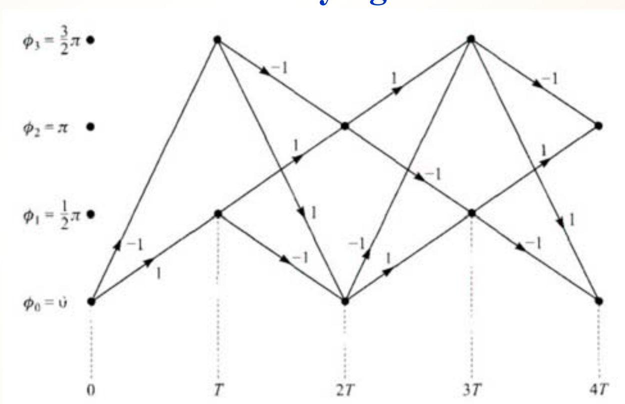
$$f_2 = f_c + \frac{1}{4T}$$

$$\Delta f = \frac{1}{2T}$$

Minimum Shift Keying (MSK)

- The spacing between the two carrier frequencies is the minimum: $\Delta f = 1/2T_s$
- Hence, it is called **minimum shift keying**

Phase changes according to the trellis

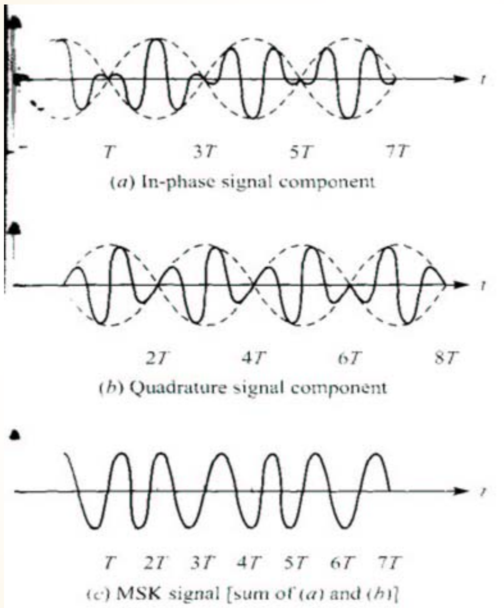


MSK

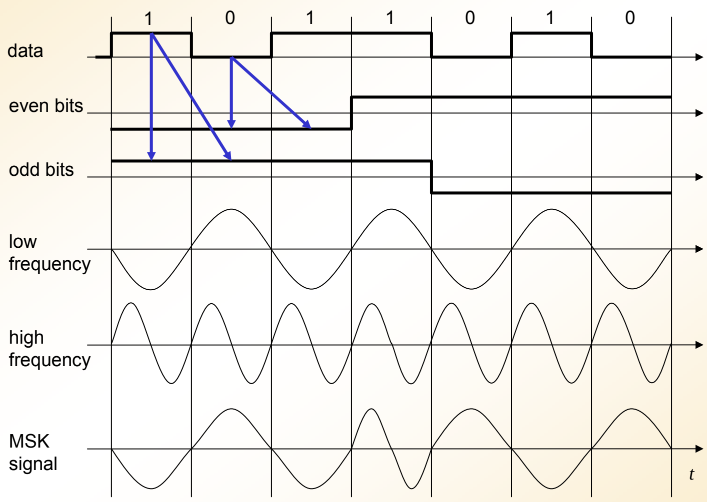
- MSK is equivalent to OQPSK with pulse shaping
- The shaping pulse is a sine wave:

  $p(t) = \sin[\pi t/(2T_s)]$,

  $0 \le t \le 2T_s$



Example of MSK



No phase shifts!

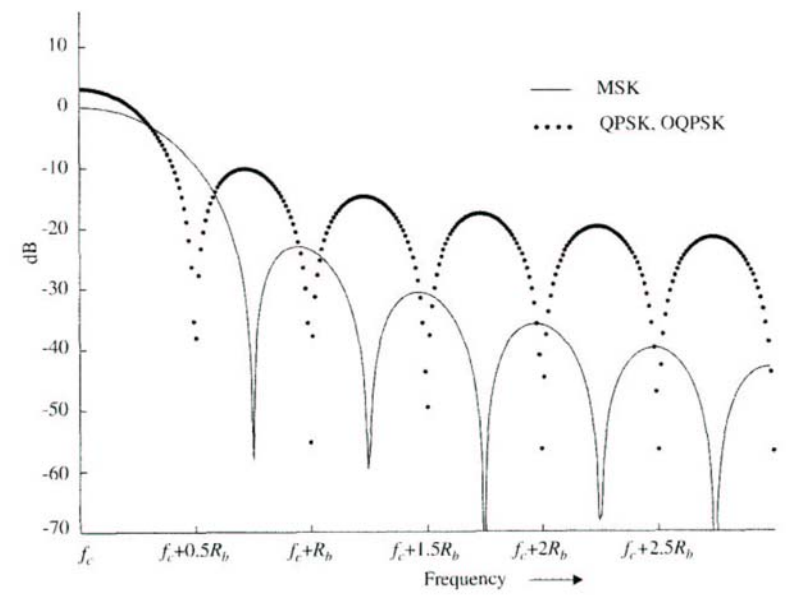
PSD of MSK

The PSD is given by:

$$P(\Delta f) = \frac{16}{\pi^2}\left(\frac{\cos 2\pi\Delta fT}{1.16\, f^2T^2}\right)^2$$

MSK has a wider first null than QPSK but lower 99%-power BW

PSD of MSK



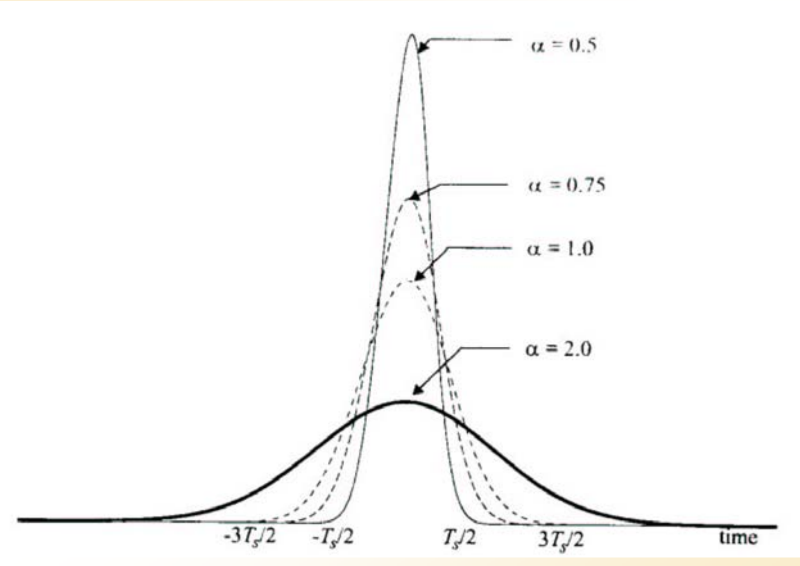
Gaussian MSK (GMSK)

- A derivative of MSK that uses a Gaussian pulse-shaping filter to filter the NRZ binary modulating waveform

$$h_G(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} t^2 \right)$$

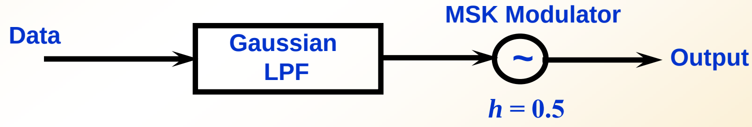
- This converts the signal from full-response CPM to a partial-response CPM signal

Gaussian Pulse



GMSK

- The Gaussian pulse-shaping filter reduces the levels of side-lobes of the GMSK spectrum compared to MSK
- It can be coherently detected (as MSK) or noncoherently detected (as FSK)



Block diagram of a GMSK transmitter using MSK modulator

GMSK

- GMSK is characterized by its BT product:
  - B is the 3-dB BW of the Gaussian filter
  - T is the symbol duration
- $B = 0.5887/\alpha$, α is the filter parameter
- As $\alpha \uparrow \Rightarrow B \downarrow \Rightarrow BT \downarrow$
  - => Sidelobe power decreases
  - => more spectral efficiency
  - => But more ISI because pulse is wider
- MSK corresponds to $\alpha = 0 \Rightarrow BT = \infty$
- In GSM, GMSK is used with $BT = 0.3$

GMSK

BT	90%	99%	99.9%	99.99%
0.2	0.52	0.79	0.99	1.22
0.25	0.57	0.86	1.09	1.37
0.5	0.69	1.04	1.33	2.08
MSK	0.78	1.2	2.76	6.00

RF Bandwidth as a fraction of R_b

Probability of Error

- Is defined as the probability of detecting a received observation as one signal given the other signal was transmitted
- It is a function of:
  - Distances between signals in the constellation (a function of the SNR)
  - Noise distribution (Usually Gaussian - AWGN)
  - Rx signal level distribution (in case of fading)
- The closer the signals in the constellation, the higher the error probability

Probability of Error

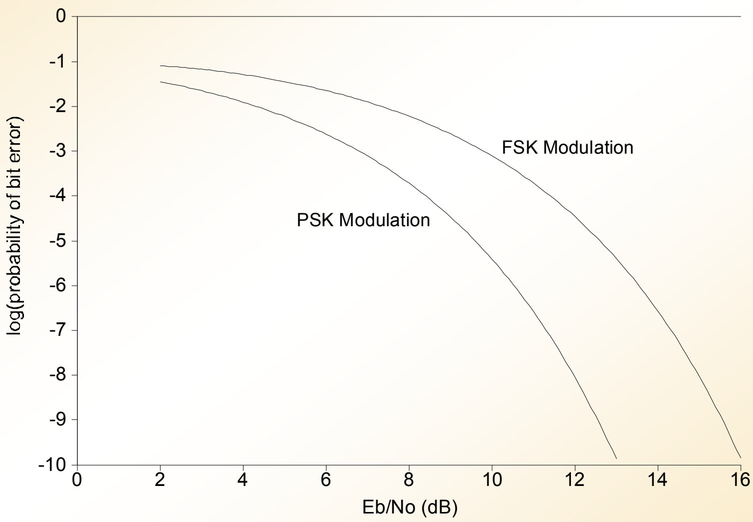
- For BPSK and QPSK demodulated by a coherent receiver, the bit error probability over an AWGN channel is:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- For non-coherent BFSK over an AWGN channel, the BER is estimated as:

$$P_b = 1/2 \exp[-0.5(E_b/N_0)]$$

Performance Over AWGN



OQPSK- Symbol Error Probability

$$P_e \approx \text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right) - \frac{1}{4}\text{erfc}^2\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e \cong \text{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

GMSK Performance

- The GMSK filtering can cause ISI if the *BT* is too small (too narrow filter and wide pulse)
- The GMSK error rate is given by:

$$P_e = Q\left(\sqrt{\frac{2\delta E_b}{N_0}}\right)$$

where $\delta = 0.68$ for $BT = 0.25$;
$\delta = 0.85$ for $BT = \infty$

Performance Over Fading Channels

- **Slow, Flat Fading:**
  - The channel varies slowly compared with the data rate; hence the channel is "constant" over one symbol duration
  - **Conditional error probability** is the error probability conditioned on the channel gain
  - The probability of error is obtained by averaging conditional error probability over the range of channel variation

Performance Over Fading Channels

$$P_e = \int_0^\infty P_e(z)p(z)dz$$

- $P_e(z)$ is the conditional probability of error with an SNR = $z = \alpha^2 E_b/N_0$
  - α is the normalized channel gain, ($<\alpha^2> = 1$)
  - E_b/N_0 is the SNR
- $p(z)$ is the probability that the SNR takes on a given value (probability density function, pdf)
- P_e = average error probability

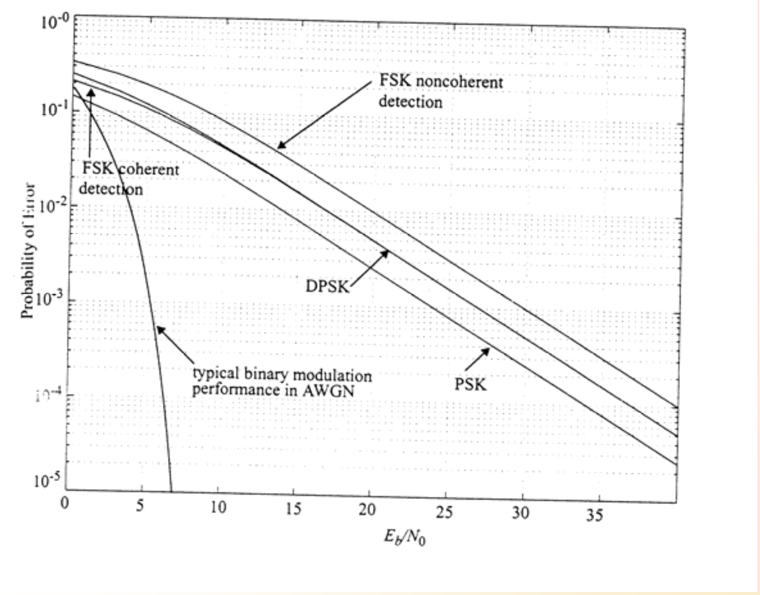
Performance Over Rayleigh Channels

- Define: $\Gamma = \langle\alpha^2\rangle E_b/N_0$ as the average SNR

For large E_b/N_0 (reasonable BER values):

- BPSK: $P_e = (4\Gamma)^{-1}$
- FSK: $P_e = (2\Gamma)^{-1}$
- NCFSK: $P_e = (\Gamma)^{-1}$
- GMSK: $P_e = (4\delta\Gamma)^{-1}$; $\delta = (2 \ln 2)^{1/2} / B$

Performance Over Rayleigh Fading



Performance Over Rayleigh Channels

- The performance is usually 20 to 40 dB **worse** than the AWGN case (no fading) for reasonable BER
- This is mostly due to deep fades which occur often because of the Rayleigh distribution
- For a Rician or Nakagami fading distributions with large *K*-factor or *m*, the error probability is smaller
- This is because the probability of deep fade is smaller
- **Mitigation Techniques:**
  - Diversity
  - Channel coding
  - Adaptive modulation

Performance Over Rayleigh Channels

- **Frequency selective** channels and Doppler spread channels have properties that restrict the channel bandwidth and, therefore, the maximum data rate.
- This causes ISI that can be scaled in terms of:

$$d = \sigma_\tau / T_s$$

- The BER hits a minimum value and cannot go lower
- This minimum BER is called the **error floor**
- **Mitigation Technique:** Equalization