Lecture 15: Modulation Techniques for Mobile Radio

Geometric Representation of Digital Modulations

Reason for Geometric Representation of Digital Modulations

The concept of digital modulation involves the following processes:

1. Obtaining the digital data from the data source
2. Creating groups of bits, a group of bits may contain one bit in binary digital modulation techniques, or M bits (such that $M > 1$) in M-ary modulation techniques
3. Using each group of bits, one of a set of 2^M pulses is selected that corresponds to the combination of bits in each group
4. Transmitting the sequence of pulses
5. Receiving the transmitted pulses
6. Separating the received pulses from each other
7. Detecting which pulse was received in each case and representing it with the corresponding bits
8. Recombining the bits into their original format (bytes for ASCII characters, 3 bytes per pixel for images, …)

Other steps may be added to the above in different digital communication systems as needed including

- Source coding/decoding (adding some redundancy bits to the transmitted data and using these redundancy bits to detect and possibly correct some of the errors that occur in the received bits)
- Equalization of the received signal (to remove the distorting effects of the channel)

Performing each of the above 8 steps is simple except for the step 7. The way of performing the process of step 7 may not be clear especially if we know that the received pulses may be very different from the transmitted pulses as a result of channel distortion and channel noise that was added to the signal as it was traveling in the channel. This means:

- If we receive a pulse that is very similar to one of the transmitted pulses, we can fairly assume that the corresponding pulse was transmitted,
- If we receive a pulse that is different from all transmitted pulses, how do we determine which pulse is the most likely one that was transmitted?

Therefore, to perform step 7 we need a method for quantifying the original transmitted pulses and the received pulses to allow for the comparison between each received pulse and all the original pulses and determining which of these is closest to the received pulse. To do this, we need to represent signals geometrically.

Concept

The concept of geometric representation of digital modulated signals requires the understanding of orthogonal signals or orthogonal pulses. Observe the following:

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- A pulse (which is a signal that is defined over a specific period of time T_S) may be a multiple of another pulse if we can represent one of them as a real value times the other. Example:

$$x_1(t) = \cos\left(2\pi f_0 t\right) \qquad 0 \le t \le T_S$$
$$x_2(t) = 5\cos\left(2\pi f_0 t\right) \qquad 0 \le t \le T_S$$

Clearly,

$$x_2(t) = 5 \cdot x_2(t) \qquad 0 \le t \le T_S$$

- A pulse may be orthogonal to another pulse if the integration of the product of the two signals over the time interval they are defined over is equal to zero. Example:

$$x_1(t) = \cos\left(2\pi f_0 t\right) \qquad 0 \le t \le T_S$$
$$x_2(t) = \sin\left(2\pi f_0 t\right) \qquad 0 \le t \le T_S$$

Where T_S is an integer multiple of $\frac{1}{f_0}$. If we multiply these two signals and integrate the product over the period $0 \le t \le T_S$ we get zero. You can easily verify that

$$\int_0^{T_S} x_1(t) \cdot x_2(t) dt = 0$$

- A pulse may neither be a multiple of some other pulse or orthogonal to it. This means that the "projection" of the first pulse on the second is non-zero. Example:

$$x_1(t) = \cos\left(2\pi f_0 t\right) \qquad 0 \le t \le T_S$$
$$x_2(t) = \cos\left(2\pi f_0 t + \frac{\pi}{4}\right) \qquad 0 \le t \le T_S$$

Where T_S is an integer multiple of $\frac{1}{f_0}$. In this case, it is clear that

$$\int_0^{T_S} x_1(t) \cdot x_2(t) dt \neq 0, \quad \text{and} \quad x_2(t) \neq c \cdot x_2(t) \quad \text{for any real constant } c$$

Given the above points, it is clear that if we represent a pulse by a vector in a Cartesian space such that:

a. The length of the vector is equivalent to the square root of the energy of the pulse (which is related to the amplitude of the pulse),
b. The angle of the vector is related to the phase of the carrier in the pulse,

then:

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1. Two pulses $x_1(t)$ and $x_2(t)$ such that $x_2(t) = c \cdot x_2(t)$ can be represented in the Cartesian space as two vectors in the same direction if c is positive or opposite direction if c is negative such that the length of the vector $\overline{x_2}$ corresponding to $x_2(t)$ is c times the length of $\overline{x_1}$ corresponding to $x_1(t)$.
2. Two pulses $x_1(t)$ and $x_2(t)$ such that $x_1(t)$ is orthogonal to $x_2(t)$ can be represented in the Cartesian space as two perpendicular vectors $\overline{x_1}$ and $\overline{x_2}$.
3. Two pulses $x_1(t)$ and $x_2(t)$ such that $x_2(t) \neq c \cdot x_2(t)$ for any real constant c, and $\int_0^{T_S} x_1(t) \cdot x_2(t) dt \neq 0$ can be represented in the Cartesian space as two vectors $\overline{x_1}$ and $\overline{x_2}$ that are neither in the same direction nor perpendicular to each other but the projection of one of them in the direction of the other is proportional to $\int_0^{T_S} x_1(t) \cdot x_2(t) dt$.

Problem Formulation

Let us define what we call a basis pulse $\phi_1(t)$ that is defined over the time period $0 \leq t \leq T_S$ such that:

$$\phi_1(t) = A \cdot \cos\left(2\pi f_c t\right)$$

and such that the **energy of the pulse is unity (1 Joul)**. This allows us to evaluate A as

$$\begin{aligned}\int_0^{T_s} \phi_1^2(t) dt &= \int_0^{T_s} A^2 \cdot \cos^2\left(2\pi f_c t\right) dt \\ &= \int_0^{T_s} \frac{A^2}{2} dt + \int_0^{T_s} \frac{A^2}{2} \cos\left(4\pi f_c t\right) dt\end{aligned}$$

Assuming that T_S is an integer multiple of $\frac{1}{f_c}$. This allows us to evaluate the above as

$$\begin{aligned}\int_0^{T_s} \phi_1^2(t) dt &= \frac{A^2 T_s}{2} + \underbrace{\int_0^{T_s} \frac{A^2}{2} \cos\left(4\pi f_c t\right) dt}_{=0} \\ &= \frac{A^2 T_s}{2}\end{aligned}$$

If we equate the above to 1, we get that

$$\frac{A^2 T_s}{2} = 1 \qquad \Rightarrow \qquad A = \sqrt{\frac{2}{T_s}}$$

which means that

$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cdot \cos\left(2\pi f_c t\right)$$

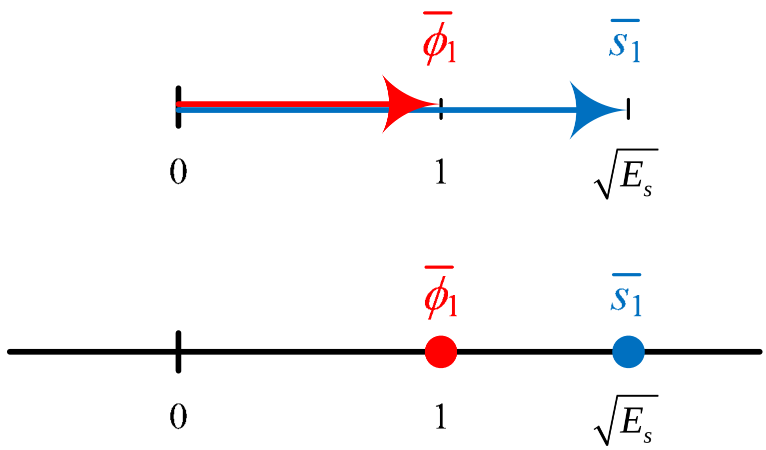
Now let a signal $s_1(t)$ which is defined over the same time period $0 \le t \le T_S$ such that

$$s_1(t) = B \cdot \cos\left(2\pi f_c t\right)$$

As done above, the energy of this pulse is $E_S = \frac{B^2 T_s}{2}$. So in terms of the energy of the pulse (also called signal or pulse) is

$$s_1(t) = \sqrt{\frac{2E_s}{T_s}} \cdot \cos\left(2\pi f_c t\right) = \sqrt{E_s}\sqrt{\frac{2}{T_s}} \cdot \cos\left(2\pi f_c t\right) = \sqrt{E_s}\phi_1(t)$$

Therefore, if we represent both $\phi_1(t)$ and $s_1(t)$ geometrically as vectors $\overline{\phi_1}$ and $\overline{s_1}$ then both vectors will be in the same direction with $\overline{\phi_1}$ having length 1 and $\overline{s_1}$ having length $\sqrt{E_s}$. Since both vectors are in the same direction, we need only one dimension to represent both of them as depicted in the following figure in two similar forms.



Now, we can represent any pulse over the time period $0 \le t \le T_S$ in terms of this basis pulse or other basis signals as will be shown later.