Lecture 12: Modulation Techniques for Mobile Radio

Amplitude Modulation Techniques

Amplitude Modulation (Full AM or Double Sideband with Carrier)

The general form for a Full AM signal (which we sometimes call DSB with Carrier or simply AM modulations) is

$$s_{AM}(t) = A_c\left[1+\mu\cdot m(t)\right]\cos(2\pi f_c t),$$

where A_c is the amplitude of the unmodulated carrier signal, $m(t)$ is the message signal with amplitude less than 1, μ is the modulation index with $0\le\mu\le 1$, and f_c is the carrier frequency in Hz. In frequency domain, the AM signal has spectrum given by

$$S_{AM}(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)+\mu\cdot M(f-f_c)+\mu\cdot M(f+f_c)\right]$$

This basically says that the full AM modulated signal contains an unmodulated carrier at the carrier frequency along with two shifted versions of the spectrum of the message signal that are shifted by the carrier frequency to the left and to the right. The following figures show representations of the message signal and the full AM modulated signal both in time and frequency domain.

For a message signal $m(t)$ with bandwidth BW equal to $BW_{m(t)}$, the bandwidth of the full AM signal (or DSBSC) signal is

$$BW_{\text{Full AM}} = 2\cdot BW_{m(t)}$$

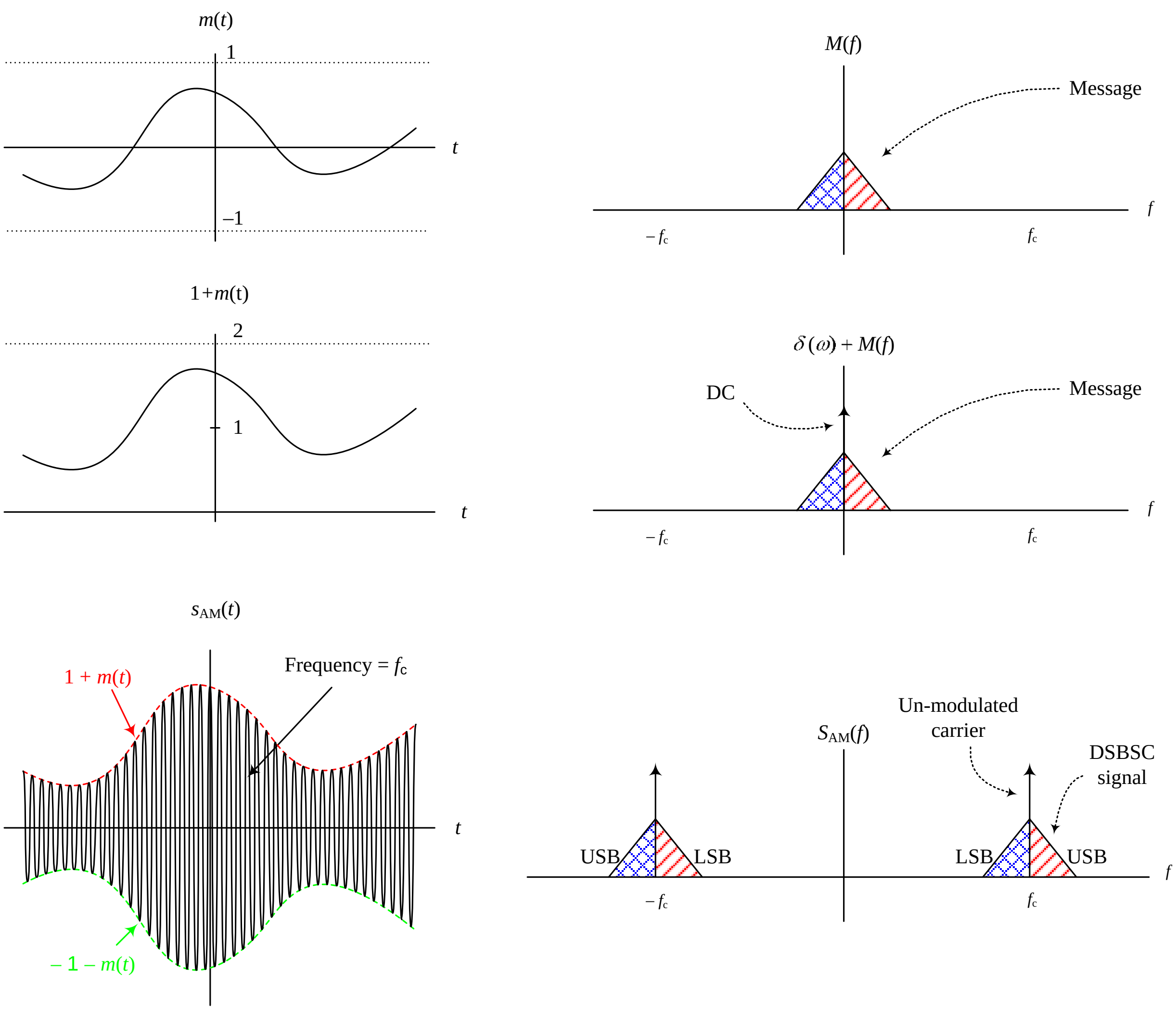
Unfortunalty, in addition to the fact that full AM is bandwidth inefficient (it uses 2 Hz of the channel to transmit each 1 Hz of the message signal) it is also power inefficient (most of the transmitted power carries no information at all (wasted in the unmodulated carrier). To see this, note that the power of the Full AM signal is

$$P_{\text{Full AM}} = \frac{1}{2}A_c^2\left[1+2\mu\left\langle m(t)\right\rangle+\mu^2\left\langle m^2(t)\right\rangle\right]$$

where $\left\langle m(t)\right\rangle$ is the average of $m(t)$ and $\left\langle m^2(t)\right\rangle$ is the power of $m(t)$. Let us assume that the message signal $m(t)$ has no DC value (zero average). This is a valid assumption for most analog information signals. In this case, the power of the full AM signal becomes

$$P_{\text{Full AM}} = \frac{1}{2}A_c^2\left[1+\mu^2\left\langle m^2(t)\right\rangle\right]$$

$$= \underbrace{\frac{1}{2}A_c^2}_{\substack{\text{Power of Unmodulated} \\ \text{Carrier (Wasted Power)}}} + \underbrace{\frac{1}{2}A_c^2\mu^2\left\langle m^2(t)\right\rangle}_{\substack{\text{Power of DSBSC Modulated} \\ \text{Signal (Useful Power)}}}$$



So, the power efficiency of the full AM signal is equal to

$$\eta_{\text{Full AM}} = \frac{\text{UsefulbPower}}{\text{Total Power}} = \frac{\text{UsefulbPower}}{\text{UsefulbPower} + \text{Wasted Power}}$$
$$= \frac{\frac{1}{2}A_c^2\mu^2\left\langle m^2(t)\right\rangle}{\frac{1}{2}A_c^2 + \frac{1}{2}A_c^2\mu^2\left\langle m^2(t)\right\rangle}$$
$$= \frac{\mu^2\left\langle m^2(t)\right\rangle}{1+\mu^2\left\langle m^2(t)\right\rangle}$$

Consider a message signal that is a sinusoid with amplitude of 1 (note that this is a practical assumption since most message signals can be represented using a finite number of sinusoids), the power of the message signal becomes equal to 0.5 and the power efficiency becomes

$$\eta_{\text{Full AM}} = \frac{\mu^2}{2+\mu^2}$$

It is clear that for $0 \le \mu \le 1$, the power efficiency has a range from 0 to 0.333. Therefore, the maximum power efficiency of full AM is only 33.3%. In most practical full AM systems, a practical value of $\mu = 0.7$ is used. In this case, the power efficiency is approximately equal to 0.2 (or 20%), which means that 80% of the transmitted power is wasted in the unmodulated carrier.

Single Sideband (SSB) Modulation

The general form for a single sideband signal is

$$s_{SSB}(t) = A_c\left[m(t)\cos(2\pi f_c t) \mp \hat{m}(t)\sin(2\pi f_c t)\right],$$

where A_c is the amplitude of the two components that make the SSB signal, $m(t)$ is the message signal, and $\hat{m}(t)$ is the Hilbert transform of the message signal. If the sign between the two terms is negative, we get an upper sideband (USB) signal, and if it is positive we get a lower sideband (LSB) signal. That is

$$s_{USB}(t) = A_c\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right]$$
$$s_{LSB}(t) = A_c\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right]$$

The Hilbert transform of the message signal is nothing but a filter that performs a -90° phase shift to each component of the signal (this is different from a delay that delays all components of the signal by a constant amount because a -90° phase shift to each component corresponds to a different delay to each frequency component of the signal). Therefore, in time domain, the Hilbert transform of $m(t)$ and the impulse response of the Hilbert transform filter are

$$\hat{m}(t) = m(t) * \frac{1}{\pi t} \qquad \Rightarrow \qquad h_{HT}(t) = \frac{1}{\pi t}$$

and in frequency domain the Hilbert transform of $m(t)$ and the transfer function of the Hilbert transform filter are

$$\hat{M}(f) = \begin{cases} jM(f) & f < 0 \\ -jM(f) & f > 0 \end{cases} \qquad \Rightarrow \qquad H_{HT}(t) = \begin{cases} j & f < 0 \\ -j & f > 0 \end{cases}$$

Notes:

1. Full AM modulation can be demodulated using coherent or non-coherent demodulation techniques and DSBSC modulation can be demodulated using coherent demodulation. For the coherent demodulation of both of these techniques, the carrier signal at the receiver must be regenerated with the same frequency an phase of the carrier of the received signal. Carrier acquisition techniques exist for generating this carrier signal locally at the receiver.
2. Carrier acquisition in the same manner used with full AM or DSBSC does not work with SSB modulation. Therefore, a very small unmodulated carrier is always transmitted with SSB signals to allow the receiver to demodulated them. This type of SSB is called **pilot tone SSB**. The level of the un-modulated carrier is very low (around 10 dB below the largest frequency component in the signal) and its purpose is only to allow the receiver to generate a carrier with the same frequency and phase as that of the received signal. The pilot tone can either be inserted in the band of the modulated signal or outside of its band.

Angle Modulation

There are two famous flavors of angle modulation: 1) Phase Modulation (PM), 2) Frequency Modulation (FM).

Frequency Modulation

The general form of a frequency modulated signal is

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right],$$

where A_c is the amplitude of the modulated signal, and k_f is the frequency deviation sensitivity (Hz/V) that determines how many Hz does the instantaneous frequency of the signal change as the message signal increases by 1 Volt.

The frequency modulation index β_f relates the message amplitude to the FM signal bandwidth:

$$\beta_f = \frac{k_f A_m}{2\pi \cdot W} = \frac{\Delta f}{W}$$

where A_m is the peak value of the message signal, W is the bandwidth of the message signal (in Hz), and Δf is peak frequency deviation of the FM signal (in Hz).

The bandwidth of the FM signal has a maximum value given by the Carson's rule given below:

$$BW_{FM} = 2\left(\beta_f + 1\right)W = 2\left(\Delta f + W\right) \qquad \text{in Hz}$$

Phase Modulation

The general form of a phase modulated signal is

$$s_{PM}(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right],$$

where A_c is the amplitude of the modulated signal, and k_p is the phase deviation sensitivity (radians/V) that determines how many radians does the phase of the signal change as the message signal increases by 1 Volt.

The phase modulation index β_p relates the message amplitude to the PM signal bandwidth:

$$\beta_p = k_p A_m = \Delta p$$

where A_m is the peak value of the message signal, and Δp is peak phase deviation of the PM signal (in Radians).