Problems on Trunking and Grade of Service

Problem 1:

Assuming that each user in a system generates a traffic intensity of 0.2 Erlangs, how many users can be supported for 0.1% probability of blocking in an Erlang B (Blocked Calls Cleared) system for a number of trunked channels (C) equal to 60.

Solution 1:

System is an Erlang B

$$A_U = 0.2 \text{ Erlangs}$$

$$\Pr\left[\text{Blocking}\right] = 0.001$$

$$C = 60 \text{ Channels}$$

From the Erlang B figure, we see that

$$A \approx 40 \text{ Erlangs}$$

Therefore,

$$U = \frac{A}{A_U} = \frac{40}{0.02} = 2000 \text{ Users}$$

Problem 2:

Assuming that each user in a system generates a traffic intensity of 0.01 Erlangs, how many users can be supported for 0.5% probability of blocking in an Erlang B (Blocked Calls Cleared) system for a number of trunked channels equal to 25.

Solution 2:

System is an Erlang B

$$A_U = 0.01 \text{ Erlangs}$$

$$\Pr\left[\text{Blocking}\right] = 0.005$$

$$C = 25 \text{ Channels}$$

From the Erlang B figure, we see that

$$A \approx 15 \text{ Erlangs}$$

Therefore,

$$U = \frac{A}{A_U} = \frac{15}{0.01} = 1500 \text{ Users}$$

Problem 3:

An Erlang B system has a number of trunked channels equal to 10. If it was found that the probability of call blocking is 0.001, and that the total number of users of the system is 300, how much time on average does each user use his phone during peak hours?

Solution 3:

System is an Erlang B

$$C = 10 \text{ Channels}$$

$$\Pr[\text{Blocking}] = 0.001$$

$$U = 300 \text{ Users}$$

To obtain the required information, we need to find the traffic intensity of the system using the Erlang B figure

$$A \approx 3.1 \text{ Erlangs}$$

Now, traffic intensity per user is

$$A_U = \frac{A_U}{U} = \frac{3.1}{300} = 0.0103 \text{ Erlangs}$$

This means that each user on average makes calls that last 1.03% of each hour during the peak hour, or

$$\text{Average Usage per User During Peak Hour} = (0.0103)\left(3600 \ \frac{\text{Seconds}}{\text{Hour}}\right)$$
$$= 37.2 \ \frac{\text{Seconds}}{\text{Hour}}$$

Problem 4:

1000 users with an average traffic intensity per user of 0.02 Erlangs cause the probability of call blocking in an Erlang B system to be is 0.05. How many channels are allocated for the system.

Solution 4:

System is an Erlang B

$$\Pr[\text{Blocking}] = 0.05$$

$$U = 1000 \text{ Users}$$

$$A_U = 0.02 \text{ Erlangs}$$

We can get the system traffic intensity using the above

$$A = A_U \cdot U = (0.02)(1000) = 20 \text{ Erlangs}$$

So, from the Erlang B chart

$$C = 25 \text{ Channels}$$

Problem 5:

A city has a population of 3 million people that are evenly distributed over an area of 1000 km^2. We know that a percentage of the population is subscribed to a cellular system. Assume that the cellular system is an **Erlang B** system with a total band of 14 MHz and full duplex channel bandwidth of 40 kHz and covers the city using hexagonal cells with radius 2 km and a cluster size of 7 cells. Assume that each user makes 1 call each 2 hours with average call duration of 1 minute and the desired probability of call blocking is 0.005. Find:

a) The total number of cells in the system
b) The number of channels per cell
c) The total number of channels in the system
d) Traffic intensity per cell
e) Maximum carried traffic for the whole system
f) The total number of users who can use the system
g) Percentage of Population of the city who can subscribe to the cellular service
h) The theoretical maximum number of users who can be served at any time.

Solution 5:

System is an Erlang B

$$\text{City Area} = 1000 \text{ km}^2$$

$$\text{Total System Bandwidth} = 14 \text{ MHz}$$

$$\text{Full Duplex Channel Bandwidth} = 40 \text{ kHz}$$

$$\text{Cells Shape} = \text{Hexagonal}$$

$$\text{Cell Radius } R = 2 \text{ km}$$

$$\text{Cluster Size } N = 7$$

$$\lambda = \frac{1 \text{ Calls}}{2 \text{ Hour}} = 0.5 \text{ Calls/Hour}$$

$$H = 1 \text{ Minute } = \frac{1}{60} \text{ Hours}$$

$$\Pr\left[\text{Blocking}\right] = 0.005$$

a) We can easily verify that the area of a hexagonal cell in terms of its radius is

$$\text{Hexagonal Cell Area } = 2.598\, R^2$$

So, the area of our cells is

$$\text{Cell Area } = 2.598\left(2\right)^2 = 10.392 \text{ km}^2$$

This gives a number of cell in the system equal to

$$\text{Number of Cells } = \frac{1000}{10.392} \approx 96 \text{ Cells}$$

b) We need to get the number of channels in the whole band first

$$\text{Number of Channels in Complete Band } = \frac{14 \text{ MHz}}{40 \text{ kHz}} = 350 \text{ Channels}$$

Dividing these channels equally among the cells of a cluster gives

$$\text{Number of Channels per Cell } = C = \frac{350}{7} = 50 \text{ Channels/Cell}$$

c) The system has 96 cells. We allocated 50 channels in each cell, so

$$\begin{aligned}\text{Total Number of Channels in the System} &= \left(50 \ \frac{\text{Channels}}{\text{Cell}}\right)\left(96 \text{ Cell}\right) \\ &= 4800 \text{ Channels}\end{aligned}$$

d) Given C and the Probability of a call being blocked (GOS), and using the Erlang B chart, we see that each cell has a traffic intensity of

$$\text{Traffic Intensity per Cell} \approx 36 \text{ Erlangs}$$

e) Maximum carried traffic over the system assumes that all cells are experiencing the maximum traffic intensity to give

$$\begin{aligned}\text{Maximum Carried Traffic over the System} &= (A)(\text{Number of Cells}) \\ &= (36 \text{ Erlangs})(96 \text{ Cells}) \\ &= 3456 \text{ Erlangs}\end{aligned}$$

f) First, let us find maximum number of Users per Cell. We need to find traffic intensity per user which is

$$\begin{aligned}\text{Traffic Intensity per User} = A_U &= \lambda \cdot H \\ &= (0.5)\left(\frac{1}{60}\right) = 0.00833 \text{ Erlangs}\end{aligned}$$

So,

$$\begin{aligned}\text{Users per cell} = U &= \frac{A}{A_U} = \left(\frac{36}{0.00833}\right) \\ &= 4322 \text{ Users/Cell}\end{aligned}$$

and

$$\begin{aligned}\text{Total Users in the Whole system} &= (4322 \text{ Users/Cell})(96 \text{ Cell}) \\ &= 414{,}912 \text{ Users}\end{aligned}$$

g) This is given by

$$\begin{aligned}\text{Percentage of City Popuation who can Subscribe} &= \frac{\text{Total Users}}{\text{City Population}} \\ &= \frac{414{,}912}{3{,}000{,}000} \\ &= 13.83\%\end{aligned}$$

h) The theoretical maximum number of users who can use the system at a particular time is equal to the total number of channels in the system (this assumes that all channels are fully occupied at a particular time)

$$\begin{aligned}&\text{Theoriticl Maximum of Users who} \\ &\text{can use System at a Particular Time} = \left(50 \ \frac{\text{Channels}}{\text{Cell}}\right)(96 \text{ Cell}) = 4800 \text{ Users}\end{aligned}$$

Problem 6: **PLEASE READ NOTE BELOW**

(This is an exercise in the book. Please read the solution there).

A hexagonal cell within a 4 cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and λ = 1 call/hour, compute the following for an Erlang C system that has a 5 % probability of delayed call:

a) How many users per square kilometer will this system support?
b) What is the probability that a delayed call will have to wait for more than 10 s?
c) What is the probability that a call will be delayed for more than 10 seconds?

Important Note:

The requested information in Part (b) is

$$\Pr\left[\text{Delay} > 20\text{s} \mid \text{The call is delayed}\right]$$

The requested information in Part (c) is

$$\Pr\left[\text{Delay} > 20\text{s}\right]$$

This is related to the first by

$$\begin{aligned}\Pr\left[\text{Delay} > 20\text{s}\right] &= \Pr\left[\text{Delay} > 20\text{s} \mid \text{The call is delayed}\right] \cdot \Pr\left[\text{The call is delayed}\right] \\ &= \Pr\left[\text{Delay} > 20\text{s} \mid \text{The call is delayed}\right] \cdot \Pr\left[\text{Delay} > 0\right] \\ &= e^{\frac{-T(C-A)}{H}} \cdot \Pr\left[\text{Delay} > 0\right]\end{aligned}$$

$$\text{where } T = 20 \text{ seconds}$$

Problem 6: (This is a modification of Problem 5 above)

Assume the system described in Problem 5 is changed to an **Erlang C** system with all parameters being the same except that now we have the probability of a **call being delayed** is 0.05 and the average call duration being 5 minutes. Find:

a) Traffic intensity per cell
b) The probability of a call getting delayed more than 3 seconds
c) Average call delay in the system

Solution 6:

System is an Erlang C

$$\text{City Area} = 1000 \text{ km}^2$$

$\text{Total System Bandwidth} = 14 \text{ MHz}$

$\text{Full Duplex Channel Bandwidth} = 40 \text{ kHz}$

$\text{Cells Shape} = \text{Hexagonal}$

$\text{Cell Radius } R = 2 \text{ km}$

$\text{Cluster Size } N = 7$

$$\lambda = \frac{1 \text{ Calls}}{2 \text{ Hour}} = 0.5 \text{ Calls/Hour}$$

$$H = 5 \text{ Minutes } = \frac{5}{60} \text{ Hours} = 300 \text{ Seconds}$$

$$\Pr\left[\text{Delay} > 0\right] = 0.05$$

a) Given C (as in previous problem) and the Probability of a call being delayed (GOS), and using the Erlang C chart, we see that each cell has a traffic intensity of

$$\text{Traffic Intensity per Cell} \approx 38 \text{ Erlangs}$$

b) The formula we use is

$$\Pr\left[\text{Delay} > T\right] = \Pr\left[\text{Delay} > 0\right] \cdot \underbrace{e^{\frac{-T(C-A)}{H}}}_{=\Pr\left[\text{Delay}>T \mid \text{Signal is Delayed (Delay>0)}\right]}$$

$$\begin{aligned}\Pr\left[\text{Delay} > 30\text{s}\right] &= \Pr\left[\text{Delay} > 0\right] \cdot e^{\frac{-3\text{s}(50-32)}{300\text{s}}} \\ &= (0.05)(0.8346) \\ &= 0.04173 \\ &= 4.173\%\end{aligned}$$

c) Now we use

$$\underbrace{D_{Avg} = \Pr\left[\text{Delay} > 0\right] \cdot \underbrace{\frac{H}{C-A}}_{=\text{Avg. Delay of Calls with Delay} > 0}}_{=\text{Avg. Delay of all Calls (delayed or not)}}$$

$$\begin{aligned}&= (0.05)\left(\frac{300\text{s}}{50\text{-}32}\right) \\ &= 0.8333 \text{ seconds}\end{aligned}$$

This is less than 1 second. But it is the average for all calls including calls that do not get delayed at all, so if you consider calls that are delayed only (with probability of 0.05 or 1 call out of each 20 gets delayed). These calls have an average delay of (300/(50-32) = 16.7 seconds).