Lecture 8: Frequency Reuse Concepts

Trunking and Grade of Service (GoS)

- Trunking is the concept that allows large number of users to use a smaller number of channels (or phone lines, customer service representatives, parking spots, public bathrooms, …) as efficiently as possible.
- It is clear that Trunking is based on statistics.
- The number of available channels in a trunked system is directly related to the probability of call blocking during peak time
- In some systems, because of high system demand, calls that cannot be initiated are
    - Blocked (caller will have to make the call later with not priority at all). Such systems are sometimes called ***Blocked Calls Cleared*** systems.
    - Queued (call is placed in a queue for several seconds until a free channel becomes available). Such systems are sometimes called ***Blocked Calls Queued*** systems.
- Trunking and Queuing theories were first studied by a mathematician called *Erlang*

What is an Erlang

One Erlang is defined as the amount of traffic intensity carrier by a channel that is completely occupied

Therefore,

1 Erlang = 1 call with a duration of 1 hour over a channel every hour

= 2 calls with a duration of 0.5 hours over the channel every hour

= 30 calls with a duration of 4 minutes over the channel every 2 hours (120 minutes)

A channel that carries 2 calls of duration 5 minutes each per hour carries (2*5 min/60 min = 1/6 Erlangs)

Grade of Service (GOS)

The grade of service (GOS) is related to the ability of a mobile phone to access the trunked mobile phone system during the busiest hour.

- To meet a specific GOS, the maximum required capacity of the system must be estimated and the proper number of channels must be allocated for the system
- GOS is a measure of the congestion of the system which is specified as the probability of a call being blocked (Erlang B system) or the probability of a call being delayed beyond a certain amount of time (Erlang C system).

Traffic Intensity

Each user in a trunked system generates a ***Traffic Intensity per User*** of A_U Erlangs given by

$$A_U = \lambda \cdot H$$

where λ = average number of call request per unit time (***Request Rate***), and

H = average duration of a call (***Holding Time***).

For a system with U users, total offered traffic intensity A is (***Offered Traffic Intensity***)

$$A = U \cdot A_U = U \cdot \lambda \cdot H$$

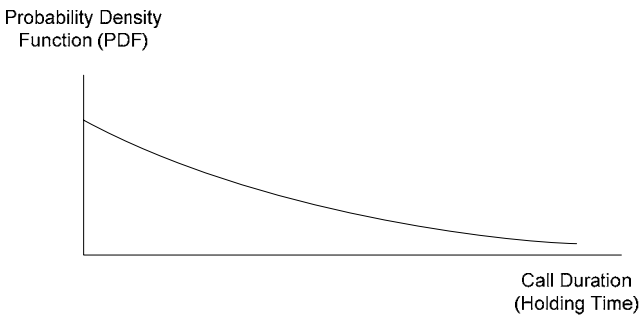
In a trunked system with C channels with traffic that is equally distributed among them, T***raffic Intensity per Channel*** A_C is given by

$$A_C = \frac{U \cdot A_U}{C} = \frac{U \cdot \lambda \cdot H}{C}$$

When offered traffic intensity (A) > Maximum capacity of system ➔ carrier traffic becomes limited due to limited capacity of the system.

To study the traffic capacity of a trunked system, we will assume the following three assumptions:

A) There are memoryless arrivals of call requests: all users including users who had blocked called may request a channel at any time. Also, because a user has just had a call blocked, does not affect his decision in making another call or the time to make that other call.
B) The probability of a user occupying a channel is exponentially distributed. So, longer calls have lower probability.



C) There are a finite number of channels available in for trunking.

Based on these assumptions, it is found that the probability of a call getting blocked in an Erlang B system is

$$\Pr[\text{Blocking}] = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C}\frac{A^k}{k!}} = \text{GOS (Erlang B)}$$

and the probability of a call getting delayed for any period of time greater than zero is

$$\Pr[\text{Delay} > 0] = \frac{A^C}{A^C + C!\left(1-\frac{A}{C}\right)\sum_{k=0}^{C-1}\frac{A^k}{k!}}$$

The probability of a call getting delayed for a period of time greater than some T is

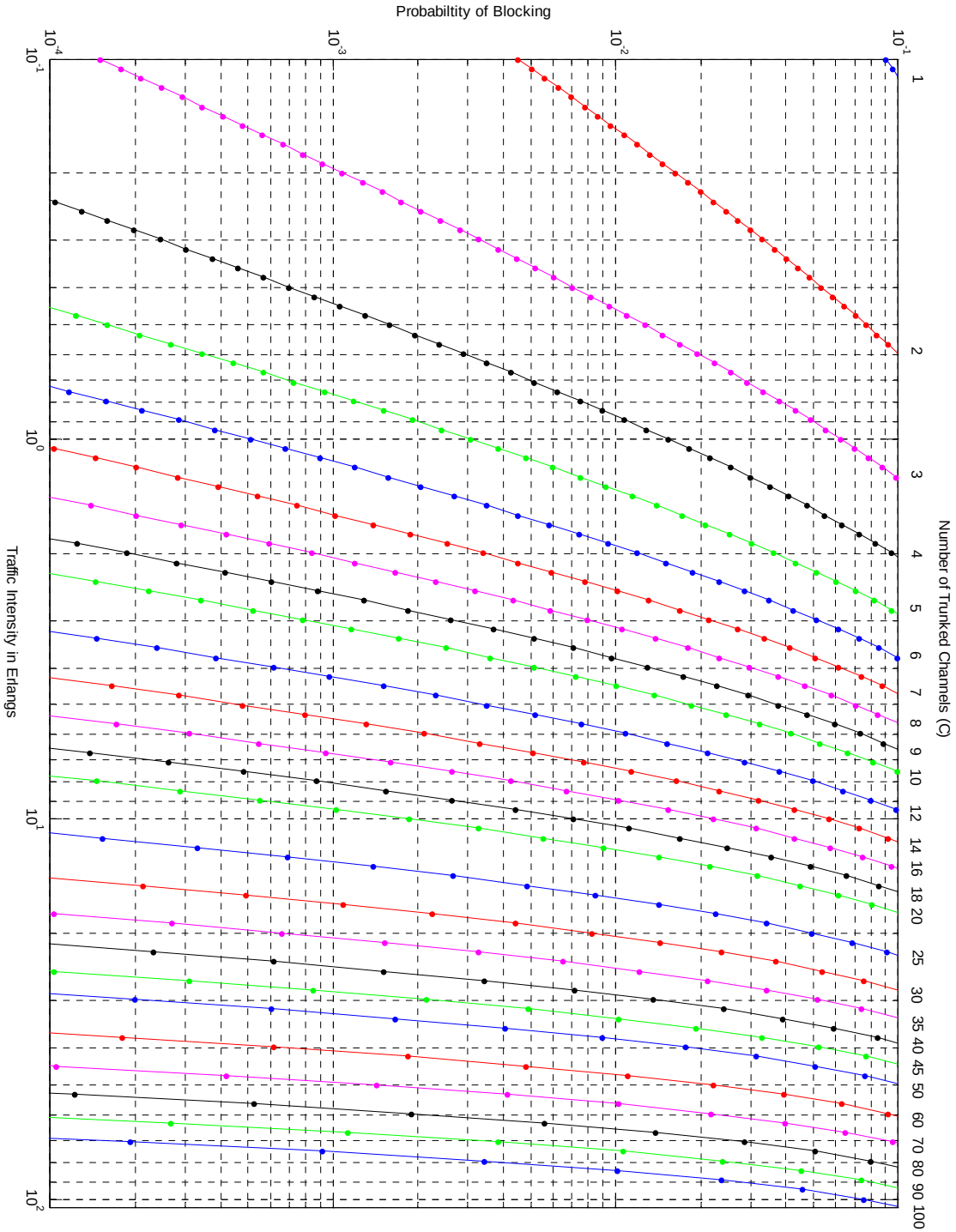
$$\Pr[\text{Delay} > T] = \Pr[\text{Delay} > 0]\cdot e^{\frac{-T(C-A)}{H}} = \text{GOS (Erlang C for a delay of length } T \text{ or longer)}$$

The average delay in this case is

$$D_{Avg} = \Pr[\text{Delay} > 0]\cdot\frac{H}{C-A}$$

The following plots are for $\Pr[\text{Blocking}]$ in an Erlang B system and the $\Pr[\text{Delay} > 0]$ in an Erlang C system for different number of trunked channels (C). These figures can be used to simplify the computations in many problems related to system capacity and GOS.

Probability of Call Blocking in an Erlang B System



Probability of Call Delay in an Erlang C System